Solving Systems of Linear Equations

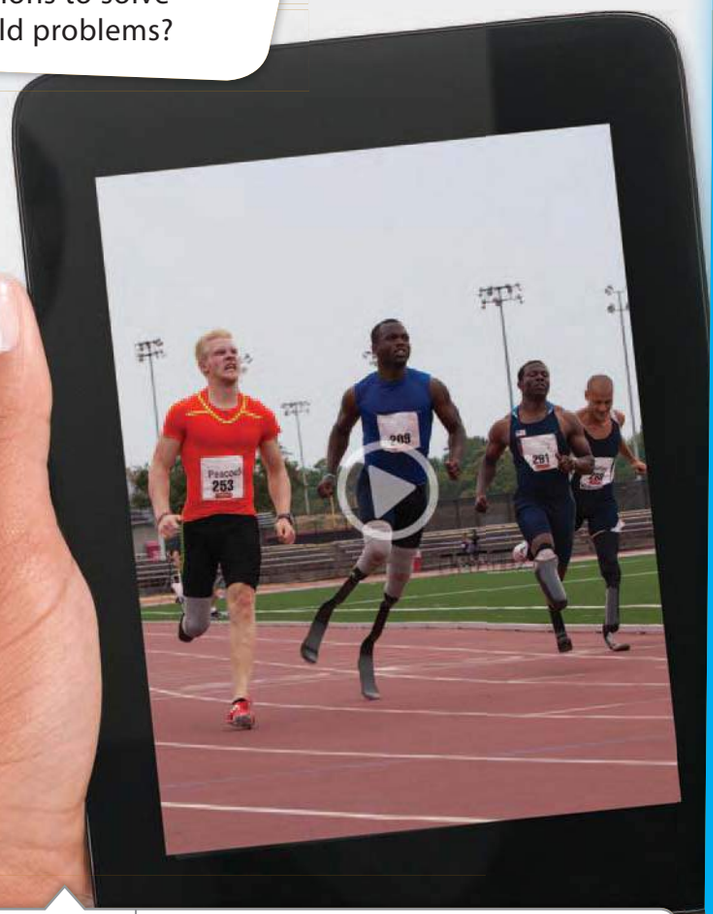
MODULE 8

CALIFORNIA

ESSENTIAL QUESTION

How can you use systems of equations to solve real-world problems?

You can use systems of linear equations to find ordered pairs where two quantities are the same, such as costs for services from two different businesses.



Real-World Video

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The distance contestants in a race travel over time can be modeled by a system of equations. Solving such a system can tell you when one contestant will overtake another who has a head start, as in a boating race or marathon.

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Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

RtI Response to Intervention

Intervention	Enrichment

Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Personal Math Trainer
Online Assessment and Intervention
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Online and Print Resources

Skills Intervention worksheets
- Skill 55 Simplify Algebraic Expressions
- Skill 64 Graph Linear Equations

Differentiated Instruction
- Challenge worksheets **PRE-AP**

Extend the Math **PRE-AP** Lesson Activities in TE

Real-World Video Viewing Guide

After students have watched the video, discuss the following:

- How can a system of equations be used to determine how long it will take two people to travel the same distance if they leave at different times?
- What does the intersection of the two graphs represent in the video? the time of arrival

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Online Practice and Help
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Simplify Algebraic Expressions

EXAMPLE Simplify $5 - 4y + 2x - 6 + y$.

$-4y + y + 2x - 6 + 5$ Group like terms.

$-3y + 2x - 1$ Combine like terms.

Simplify.

1. $14x - 4x + 21$ $10x + 21$
2. $-y - 4x + 4y$ $3y - 4x$
3. $5.5a - 1 + 21b + 3a$ $8.5a + 21b - 1$
4. $2y - 3x + 6x - y$ $y + 3x$

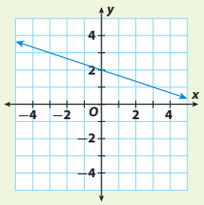
Graph Linear Equations

EXAMPLE Graph $y = -\frac{1}{3}x + 2$.

Step 1: Make a table of values.

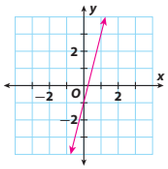
x	$y = -\frac{1}{3}x + 2$	(x, y)
0	$y = -\frac{1}{3}(0) + 2 = 2$	(0, 2)
3	$y = -\frac{1}{3}(3) + 2 = 1$	(3, 1)

Step 2: Plot the points.
Step 3: Connect the points with a line.

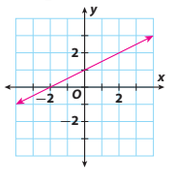


Graph each equation.

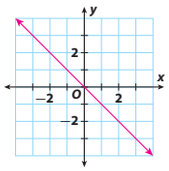
5. $y = 4x - 1$



6. $y = \frac{1}{2}x + 1$



7. $y = -x$



PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores the concept of systems of equations in an actual eighth-grade classroom.

Professional Development
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Interactive Whiteboards
Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.

Reading Start-Up EL

Have students complete the activities on this page by working alone or with others.

Strategies for English Learners

Each lesson in the TE contains specific strategies to help English Learners of all levels succeed.

Emerging: Students at this level typically progress very quickly, learning to use English for immediate needs as well as beginning to understand and use academic vocabulary and other features of academic language.

Expanding: Students at this level are challenged to increase their English skills in more contexts, and learn a greater variety of vocabulary and linguistic structures, applying their growing language skills in more sophisticated ways appropriate to their age and grade level.

Bridging: Students at this level continue to learn and apply a range of high-level English language skills in a wide variety of contexts, including comprehension and production of highly technical texts.

Active Reading

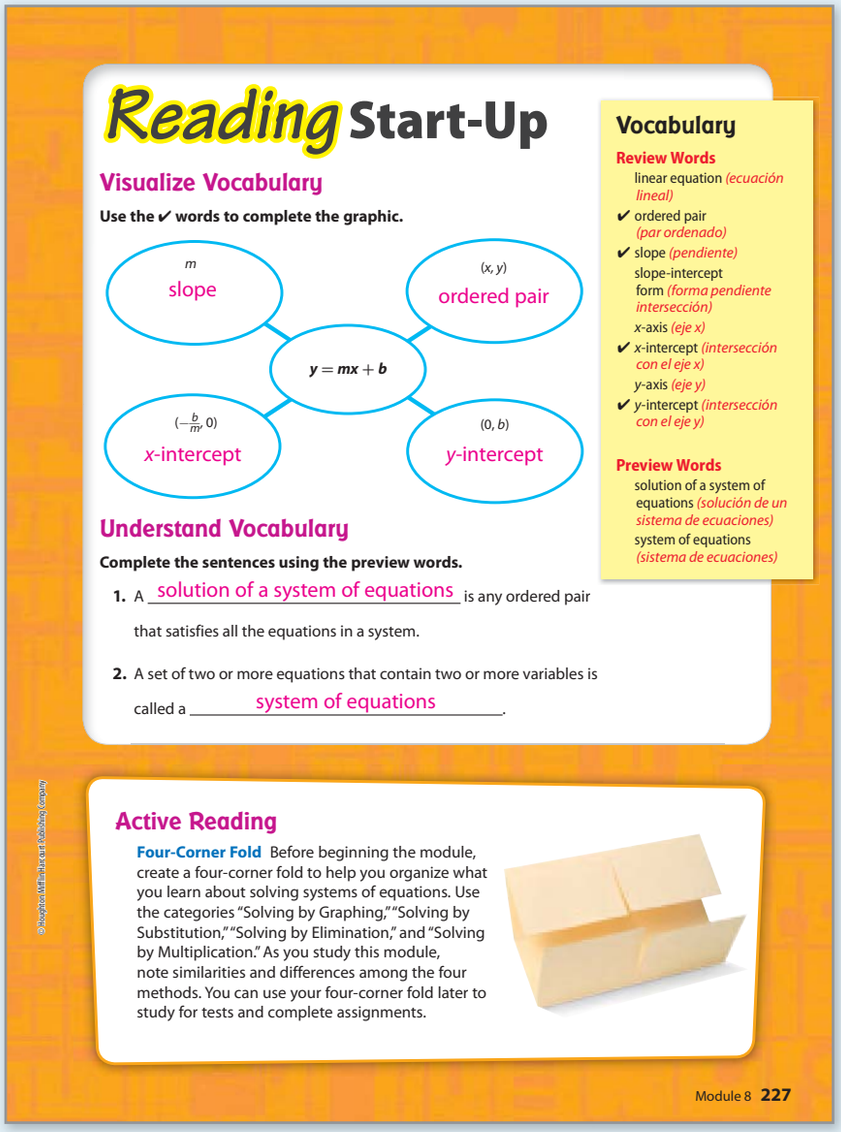
Integrating Language Arts

Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

Additional Resources

Differentiated Instruction

- Reading Strategies EL



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic.

Understand Vocabulary

Complete the sentences using the preview words.

1. A solution of a system of equations is any ordered pair that satisfies all the equations in a system.
2. A set of two or more equations that contain two or more variables is called a system of equations.

Vocabulary

Review Words

linear equation *(ecuación lineal)*
✔ ordered pair *(par ordenado)*
✔ slope *(pendiente)*
slope-intercept form *(forma pendiente intersección)*
x-axis *(eje x)*
✔ *x*-intercept *(intersección con el eje x)*
y-axis *(eje y)*
✔ *y*-intercept *(intersección con el eje y)*

Preview Words

solution of a system of equations *(solución de un sistema de ecuaciones)*
system of equations *(sistema de ecuaciones)*

Active Reading

Four-Corner Fold Before beginning the module, create a four-corner fold to help you organize what you learn about solving systems of equations. Use the categories "Solving by Graphing," "Solving by Substitution," "Solving by Elimination," and "Solving by Multiplication." As you study this module, note similarities and differences among the four methods. You can use your four-corner fold later to study for tests and complete assignments.

CA CC Focus | Coherence | Rigor

Tracking Your Learning Progression

Before	In this module	After
Students understand: • how to solve linear equations • how to graph linear equations	Students will learn how to: • solve systems of two linear equations in two variables using graphing, elimination, and substitution • analyze special systems that have no solution or an infinite number of solutions • represent real-world situations using systems of equations	Students will connect: • graphical and algebraic representations of systems of equations and their solutions

GETTING READY FOR

Solving Systems of Linear Equations

Use the exercises on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

CA Common Core Standards

Content Areas

CA CC Expressions and Equations—8.EE

Cluster Analyze and solve linear equations and pairs of simultaneous linear equations.

Go online to see a complete unpacking of the CA Common Core Standards.

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GETTING READY FOR

Solving Systems of Linear Equations

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

CA CC 8.EE.8a

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Key Vocabulary

solution of a system of equations *(solución de un sistema de ecuaciones)* A set of values that make all equations in a system true.

system of equations *(sistema de ecuaciones)* A set of two or more equations that contain two or more variables.

What It Means to You

You will understand that the points of intersection of two or more graphs represent the solution to a system of linear equations.

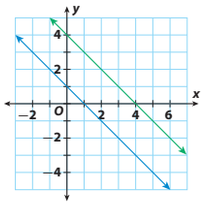
EXAMPLE 8.EE.8a, 8.EE.8b

Use the elimination method.

A. $-x = -1 + y$

$x + y = 4$

$y = y + 3$



This is never true, so the system has no solution. The graphs never intersect.

Use the substitution method.

B. $2y + x = 1$

$y - 2 = x$

$2y + (y - 2) = 1$

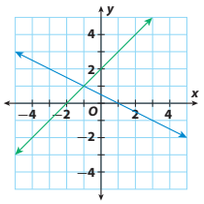
$3y - 2 = 1$

$y = 1$

$x = y - 2$

$x = 1 - 2$

$x = -1$



Only one solution: $x = -1$, $y = 1$. The graphs intersect at the point $(-1, 1)$.

Use the multiplication method.

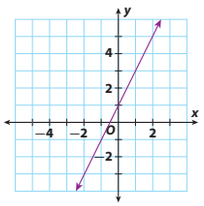
C. $3y - 6x = 3$

$y - 2x = 1$

$3y - 6x = 3$

$3y - 6x = 3$

$0 = 0$



This is always true. So the system has infinitely many solutions. The graphs are the same line.

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228 Unit 3

California Common Core Standards	Lesson 8.1	Lesson 8.2	Lesson 8.3	Lesson 8.4	Lesson 8.5
CA CC 8.EE.8 Analyze and solve pairs of simultaneous linear equations.	✓				
CA CC 8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	✓				
CA CC 8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.		✓	✓	✓	✓
CA CC 8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.	✓	✓	✓	✓	✓

Lesson Support

Content Objective Students will learn how to solve a system of equations by graphing.

Language Objective Students will demonstrate how to solve a system of equations by graphing.

California Common Core Standards

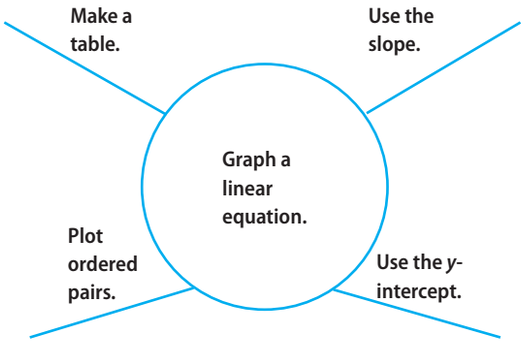
CA CC **8.EE.8a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

CA CC **MP.3** Construct viable arguments and critique the reasoning of others.

CA CC **Focus | Coherence | Rigor**

Building Background

Eliciting Prior Knowledge Have students create an information wheel to write the tools they can use to graph a linear equation. They should include as graphing tools the slope and *y*-intercept of a linear equation in slope-intercept form, making a table with at least two ordered pairs, and plotting the ordered pairs in the coordinate plane.



Learning Progressions

In this lesson, students solve a system of two linear equations by graphing the equations and identifying the point of intersection of the lines. Important understandings for students include the following:

- Solve a system of linear equations by graphing.
- Rewrite a linear equation written in standard form in slope-intercept form.
- Solve problems using systems of linear equations graphically.

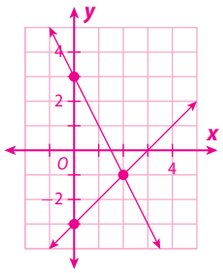
In Grade 8, students start to solve problems that lead to simultaneous equations. This lesson introduces students to the graphical method that can be used to solve a system. It introduces students to real-world problem situations that are solved by formulating and then solving a linear system. The emphasis on context helps students recognize the usefulness of setting up and solving these systems.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Analyze and solve linear equations and pairs of simultaneous linear equations.*** Ask students whether the ordered pair (2, 1) is a solution of the following system:

$$\begin{cases} x - y = 3 \\ 2x + y = 3 \end{cases}$$

After they test the point in the equations, ask them to solve the system by graphing. no; (2, −1)



Language Support

California ELD Standards

CA ELD **Emerging 2.I.6c. Reading/viewing closely** – Use knowledge of morphology, context, reference materials, and visual cues to determine the meanings of unknown and multiple-meaning words on familiar topics.

CA ELD **Expanding 2.I.6c. Reading/viewing closely** – Use knowledge of morphology, context, reference materials, and visual cues to determine the meanings, including figurative and connotative meanings, of unknown and multiple-meaning words on a variety of new topics.

CA ELD **Bridging 2.I.6c. Reading/viewing closely** – Use knowledge of morphology, context, reference materials, and visual cues to determine the meanings, including figurative and connotative meanings, of unknown and multiple-meaning words on a variety of new topics.

Linguistic Support

Academic/Content Vocabulary

Demonstrate for English learners to help them better understand systems of equations.

Have students model a problem that involves two objects moving at different speeds. Beginning with two students at the same location, have one walk forward at a slow, steady pace. A few seconds later, have the other student walk along the same path at a faster speed until the second student catches up with the first student. Ask whether the second person could catch up if he or she walked the same speed, or slower speed. The point at which the second person catches up models the point at which the two lines intersect.

Cognates and Borrowed Words

Students will be using the word *equation* frequently in algebra. *Equation* is a cognate with Spanish. English and Spanish share many words from Latin, so they share many roots, prefixes, and suffixes. Many words in English that end with *-tion* are shared with Spanish words ending in *-ción*. These shared words are known as cognates. Some are shown below.

English *-tion*	Spanish *-ción*
multiplication	multiplicación
equation	ecuación
solution	solución
elimination	eliminación

Leveled Strategies for English Learners

Emerging In order to make the concept easier to grasp, have students work in small teams to select a problem from this lesson, role play, and show the solution.

Expanding Have students plan a party that must stay within a budget. Pairs of students will create a system of linear equations that represents how a certain amount of money will be spent on two items. For example, one pair works with pizza and drinks and another with plates and cups.

Bridging Have students observe as another group plans a party that must stay within a budget. Students prepare to report on the steps that were taken to plan the party so that it would stay on budget.

Math Talk To help students answer the question posed in Math Talk, provide sentence frames for them to use.

LESSON

8.1 Solving Systems of Linear Equations by Graphing

CA Common Core Standards

The student is expected to:

CA CC **Expressions and Equations—8.EE.8a**

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
Also 8.EE.8, 8.EE.8c

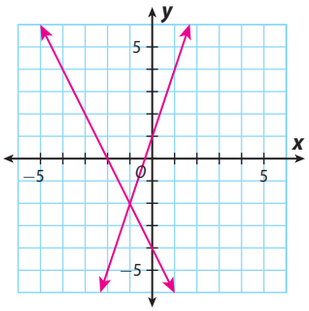
Mathematical Practices

CA CC **MP.3** Logic

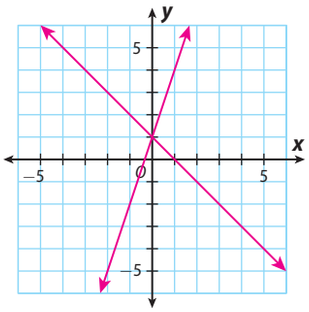
ADDITIONAL EXAMPLE 1

Solve each system by graphing.

A $\begin{cases} y = -2x - 4 \\ y = 3x + 1 \end{cases}$ $(-1, -2)$



B $\begin{cases} y = -x + 1 \\ y = 3x + 1 \end{cases}$ $(0, 1)$



Interactive Whiteboard
Interactive example available online

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Engage

ESSENTIAL QUESTION

How can you solve a system of equations by graphing? Sample answer: Find the ordered pair at the point where the graphs of the equations intersect.

Motivate the Lesson

Ask: Do you think you could catch up with your friend before he gets home if you leave school 10 minutes later but travel twice as fast? Begin the Explore Activity to see how you can solve an algebraic version of a problem like this.

Explore

EXPLORE ACTIVITY

Focus on Reasoning

Elicit from students that the solution is the ordered pair represented by the point where the two lines intersect.

Explain

EXAMPLE 1

Connect Vocabulary EL

A *system* is a set of things working together. If any one of the things is not working, then the system does not work. Similarly, an ordered pair is the solution of a system of equations only if it is the solution for each equation.

Questioning Strategies Mathematical Practices

- Why is the point of intersection a solution? It is the only ordered pair whose *x*- and *y*-values satisfy both equations.
- How might you be able to tell from the given system in part B that the lines intersect at (0, −3)? Both equations have the intercept −3, so they both pass through the point (0, −3).

Integrating Language Arts EL

Encourage English learners to take notes on new terms or concepts and to write them in familiar language.

LESSON 8.1

Solving Systems of Linear Equations by Graphing

CA CC 8.EE.8a

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. *Also 8.EE.8, 8.EE.8c*

ESSENTIAL QUESTION

How can you solve a system of equations by graphing?

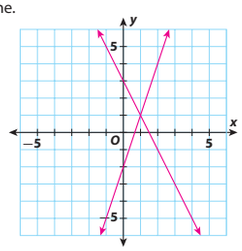
EXPLORE ACTIVITY CA CC 8.EE.8a

Investigating Systems of Equations

You have learned several ways to graph a linear equation in slope-intercept form. For example, you can use the slope and y-intercept or you can find two points that satisfy the equation and connect them with a line.

Slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

A Graph the pair of equations together: $\begin{cases} y = 3x - 2 \\ y = -2x + 3 \end{cases}$.



B Explain how to tell whether $(2, -1)$ is a solution of the equation $y = 3x - 2$ without using the graph.

Substituting $(2, -1)$ into the equation results in a false statement, so it is not a solution.

C Explain how to tell whether $(2, -1)$ is a solution of the equation $y = -2x + 3$ without using the graph.

Substituting $(2, -1)$ into the equation results in a true statement, so it is a solution.

D Use the graph to explain whether $(2, -1)$ is a solution of each equation.

If $(2, -1)$ is on the line, it is a solution. So, $(2, -1)$ is not a solution of $y = 3x - 2$ but it is a solution of $y = -2x + 3$.

E Determine if the point of intersection is a solution of both equations.

Point of intersection: $(1, 1)$

$y = 3x - 2$: $1 = 3(1) - 2$, $1 = 1$

$y = -2x + 3$: $1 = -2(1) + 3$, $1 = 1$

The point of intersection **is** / **is not** the solution of both equations.

Math On the Spot
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My Notes

Solving Systems Graphically

An ordered pair (x, y) is a solution of an equation in two variables if substituting the x- and y-values into the equation results in a true statement. A **system of equations** is a set of equations that have the same variables. An ordered pair is a **solution of a system of equations** if it is a solution of every equation in the set.

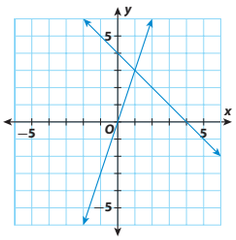
Since the graph of an equation represents all ordered pairs that are solutions of the equation, if a point lies on the graphs of two equations, the point is a solution of both equations and is, therefore, a solution of the system.

EXAMPLE 1 CA CC 8.EE.8

Solve each system by graphing.

A $\begin{cases} y = -x + 4 \\ y = 3x \end{cases}$

STEP 1 Start by graphing each equation.



STEP 2 Find the point of intersection of the two lines. It appears to be (1, 3). Substitute to check if it is a solution of both equations.

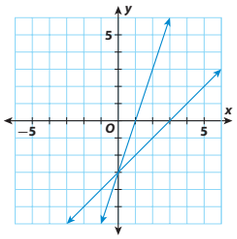
$y = -x + 4$: $3 \stackrel{?}{=} -(1) + 4$, $3 = 3$ ✓

$y = 3x$: $3 \stackrel{?}{=} 3(1)$, $3 = 3$ ✓

The solution of the system is (1, 3).

B $\begin{cases} y = 3x - 3 \\ y = x - 3 \end{cases}$

STEP 1 Start by graphing each equation.



STEP 2 Find the point of intersection of the two lines. It appears to be (0, −3). Substitute to check if it is a solution of both equations.

$y = 3x - 3$: $-3 \stackrel{?}{=} 3(0) - 3$, $-3 = -3$ ✓

$y = x - 3$: $-3 \stackrel{?}{=} 0 - 3$, $-3 = -3$ ✓

The solution of the system is (0, −3).

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.3

This lesson provides an opportunity to address this Mathematical Practices standard. It calls for students to reason logically about what it means for a system of two linear equations in two variables to have a unique solution, both graphically and algebraically. Students also have the opportunity to make conjectures about the conditions under which a system of three linear equations in two variables will have a unique solution.

Math Background

Solving a system of equations by graphing has a limitation that algebraic methods do not. Only an algebraic method such as substitution or elimination (which students will learn later) can determine the exact solution to a system. Graphical solutions are considered approximate. This is evident if the solution is not a lattice point where grid lines meet.

YOUR TURN

Focus on Modeling Mathematical Practices

Review methods of graphing lines. Students can either use a table of values or plot the *y*-intercept and use the slope to plot additional points on the line. Explain that if either line is not drawn accurately, the solution point will not be correct.

EXAMPLE 2

Questioning Strategies Mathematical Practices

- Which line represents the amount of money spent? How do you know? The steeper line is the money spent because when $0 is spent on hot dogs, $11 is spent on drinks.
- What would you do if either *x* or *y* appeared to be a number that is not an integer? Recheck my work because you cannot buy part of a soda or part of a hot dog.

Avoid Common Errors

Make sure that students understand that, in order to graph an equation using the slope and *y*-intercept, they must first rewrite the equation in slope-intercept form.

ADDITIONAL EXAMPLE 2

Zander uses an on-line store that sells songs for $1 each and movies for $6. He used all of his $20 allowance to buy 10 items. How many songs and how many movies did he buy? 8 songs and 2 movies

Interactive Whiteboard
Interactive example available online

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Animated Math
Explore a System of Equations

Using a model of two runners, students discover how a graph can show where the runners' paths will intersect.

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Reflect

1. **What If?** If you want to include another linear equation in the system of equations in part A of Example 1 without changing the solution, what must be true about the graph of the equation?

   The graph must be a line that passes through the point (1, 3).

2. **Analyze Relationships** Suppose you include the equation $y = -2x - 3$ in the system of equations in part B of Example 1. What effect will this have on the solution of the system? Explain your reasoning.

   Sample answer: The solution doesn't change. The equations all have the same y-intercept -3, and the graphs intersect at $(0, -3)$.

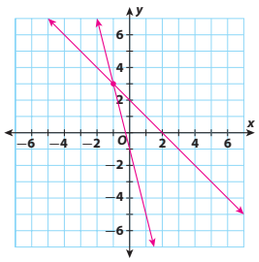
YOUR TURN

Solve each system by graphing. Check by substitution.

3. $\begin{cases} y = -x + 2 \\ y = -4x - 1 \end{cases}$ $(-1, 3)$

Check:

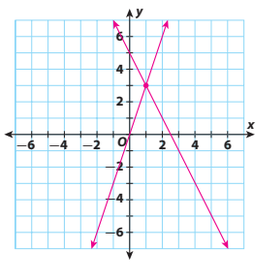
$y = -x + 2$	$y = -4x - 1$
$3 = -(-1) + 2$	$3 = -4(-1) - 1$
$3 = 3$ ✓	$3 = 3$ ✓



4. $\begin{cases} y = -2x + 5 \\ y = 3x \end{cases}$ $(1, 3)$

Check:

$y = -2x + 5$	$y = 3x$
$3 = -2(1) + 5$	$3 = 3(1)$
$3 = 3$ ✓	$3 = 3$ ✓



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Solving Problems Using Systems of Equations

When using graphs to solve a system of equations, it is best to rewrite both equations in slope-intercept form for ease of graphing.

To write an equation in slope-intercept form starting from $ax + by = c$:

$ax + by = c$

$by = c - ax$	Subtract ax from both sides.
$y = \frac{c}{b} - \frac{ax}{b}$	Divide both sides by b.
$y = -\frac{a}{b}x + \frac{c}{b}$	Rearrange the equation.

EXAMPLE 2 Real World CA CC 8.EE.8c, 8.EE.8

Keisha and her friends visit the concession stand at a football game. The stand charges \$2 for a sandwich and \$1 for a lemonade. The friends buy a total of 8 items for \$11. Tell how many sandwiches and how many lemonades they bought.

STEP 1 Let x represent the number of sandwiches they bought and let y represent the number of lemonades they bought.

Write an equation representing the **number of items they purchased.**

Number of sandwiches + Number of lemonades = Total items

$x + y = 8$

Write an equation representing the **money spent on the items.**

Cost of 1 sandwich times number of sandwiches + Cost of 1 lemonade times number of lemonades = Total cost

$2x + 1y = 11$

STEP 2 Write the equations in slope-intercept form. Then graph.

$x + y = 8$

$y = 8 - x$

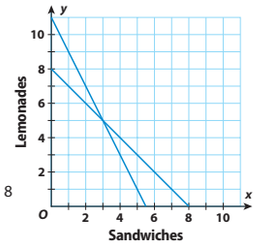
$y = -x + 8$

$2x + 1y = 11$

$1y = 11 - 2x$

$y = -2x + 11$

Graph the equations $y = -x + 8$ and $y = -2x + 11$.



DIFFERENTIATE INSTRUCTION

Home Connection

Have the whole class plan a party that must stay within a budget. Pairs of students will create a system of linear equations that represents how a certain amount of money will be spent on two items. For example, one pair works with pizza and soda, another with plates and cups.

Kinesthetic Experience

Have students model a problem that involves two objects moving at different speeds. Have two students stand at the same location and have one student walk forward at a slow, steady pace. A few seconds later, have another student walk along the same path at a faster speed until the second student catches up with the first.

Ask whether the second person could catch up if they walked the same speed or slower. The point at which they catch up is the point at which they intersect. In other words, the moment when the elapsed time and the distance traveled is the same for both.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

YOUR TURN

Engage with the Whiteboard

After graphing both lines, discuss what each line means. Point out that the graph of $2x + 4y = 20$ shows how many of each game can be played for \$20 with no restriction on the number of games. Draw graphs of $x + y = 5$ and $x + y = 7$ to see how the solution of the system changes with a different total number of games played.

Elaborate

Talk About It

Summarize the Lesson

Ask: Given a coordinate plane that contains the graph of two linear equations, how can you identify the solution to the system of equations? The solution is the ordered pair at the point of intersection because the *x*- and *y*-values in the ordered pair will make both equations true.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 2, replace the second equation with $y = x - 5$ and have a student graph this equation. Have students describe the solution to the new system.

Avoid Common Errors

Exercises 1–3 Remind students to check their answers algebraically after finding the solution on the graph.

Exercise 2 Remind students that a linear equation in two variables has to be in slope-intercept form before they can read the slope and *y*-intercept from the coefficient of *x* and the constant term.

STEP 3 Use the graph to identify the solution of the system of equations. Check your answer by substituting the ordered pair into both equations.

Apparent solution: (3, 5)
Check:

$x + y = 8$ $\quad$ $2x + y = 11$

$3 + 5 \stackrel{?}{=} 8$ $\quad$ $2(3) + 5 \stackrel{?}{=} 11$

$8 = 8$ ✓ $\quad$ $11 = 11$ ✓

The point (3, 5) is a solution of both equations.

STEP 4 Interpret the solution in the original context.

Keisha and her friends bought 3 sandwiches and 5 lemonades.

Reflect

5. **Conjecture** Why do you think the graph is limited to the first quadrant?

It would not make sense to buy a negative number of items or to spend a negative amount of money.

YOUR TURN

6. During school vacation, Marquis wants to go bowling and to play laser tag. He wants to play 6 total games but needs to figure out how many of each he can play if he spends exactly $20. Each game of bowling is $2 and each game of laser tag is $4.

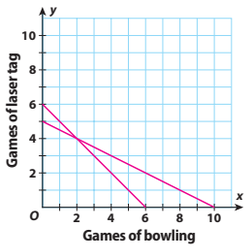
a. Let x represent the number of games Marquis bowls and let y represent the number of games of laser tag Marquis plays. Write a system of equations that describes the situation. Then write the equations in slope-intercept form.

$x + y = 6$ and $2x + 4y = 20$; $y = -x + 6$ and $y = -0.5x + 5$

b. Graph the solutions of both equations.

c. How many games of bowling and how many games of laser tag will Marquis play?

Marquis will bowl 2 games and play 4 games of laser tag.



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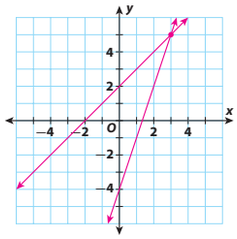
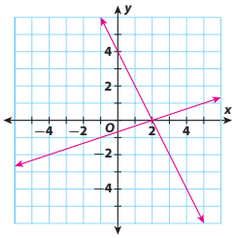
Guided Practice

Solve each system by graphing. (Examples 1 and 2)

1. $\begin{cases} y = 3x - 4 \\ y = x + 2 \end{cases}$ (3, 5)

2. $\begin{cases} 2x + y = 4 \\ -3x + 9y = -6 \end{cases}$ (2, 0)





3. **Mrs. Morales wrote a test with 15 questions covering spelling and vocabulary. Spelling questions (x) are worth 5 points and vocabulary questions (y) are worth 10 points. The maximum number of points possible on the test is 100.** (Example 2)

a. Write an equation in slope-intercept form to represent the number of questions on the test.

$y = -x + 15$

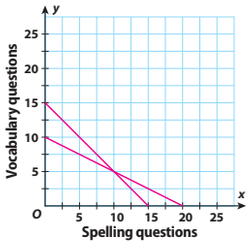
b. Write an equation in slope-intercept form to represent the total number of points on the test.

$y = -0.5x + 10$

c. Graph the solutions of both equations.

d. Use your graph to tell how many of each question type are on the test.

10 spelling questions and 5 vocabulary questions



ESSENTIAL QUESTION CHECK-IN

4. When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?

Every point on a line makes a linear equation true. A point that is on both lines (the intersection point) makes both equations true.

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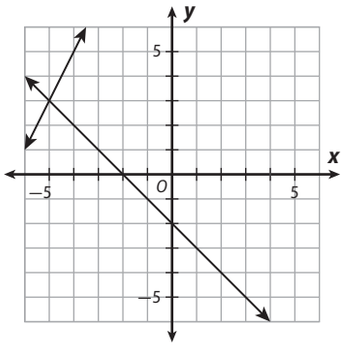
8.1 LESSON QUIZ

CA CC **8.EE.8a, 8.EE.8c**

1. What is the solution to the system of equations shown?



Diego works at a warehouse that ships two types of packages, a red package weighing 4 pounds and a blue package weighing 6 pounds. Diego shipped a total of 40 packages weighing 180 pounds.

2. Graph the system of linear equations.

3. How many red and blue packages were in the shipment?

Lesson Quiz available online

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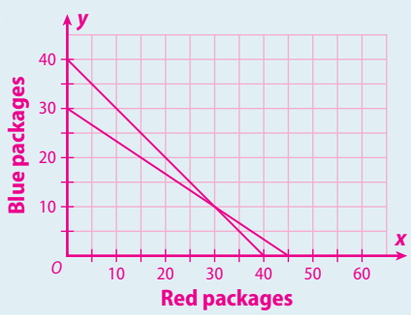
Answers

1. (–5, 3)

2.



3. 30 red packages and 10 blue packages

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.EE.8, 8.EE.8a, 8.EE.8c**

Concepts & Skills	Practice
Explore Activity Investigating Systems of Equations	Exercises 1–2
Example 1 Solving Systems Graphically	Exercises 1–3, 6–7
Example 2 Solving Problems Using Systems of Equations	Exercises 2, 3, 6–8

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
5	**1** Recall	**MP.6** Precision
6–7	**2** Skills/Concepts	**MP.2** Reasoning
8–9	**2** Skills/Concepts	**MP.4** Modeling
10	**3** Strategic Thinking H.O.T.	**MP.4** Modeling
11	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

CLUSTER CONNECTION

Exercise 7 combines concepts from the California Common Core cluster "Analyze and solve linear equations and pairs of simultaneous linear equations."

Name _____ Class _____ Date _____

8.1 Independent Practice

CA CC 8.EE.8, 8.EE.8a, 8.EE.8c

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5. Vocabulary A system of equations is a set of equations that have the same variables.

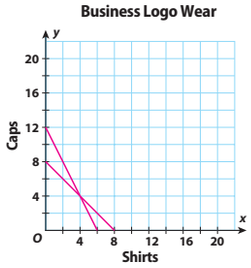
6. Eight friends started a business. They will wear either a baseball cap or a shirt imprinted with their logo while working. They want to spend exactly $36 on the shirts and caps. Shirts cost $6 each and caps cost $3 each.

a. Write a system of equations to describe the situation. Let x represent the number of shirts and let y represent the number of caps.

$x + y = 8$

$6x + 3y = 36$

b. Graph the system. What is the solution and what does it represent?



The solution is (4, 4). It represents that 4 people will get shirts and 4 people will get caps.

7. Multistep The table shows the cost for bowling at two bowling alleys.

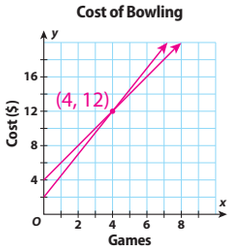
	Shoe Rental Fee	Cost per Game
Bowl-o-Rama	$2.00	$2.50
Bowling Pinz	$4.00	$2.00

a. Write a system of equations, with one equation describing the cost to bowl at Bowl-o-Rama and the other describing the cost to bowl at Bowling Pinz. For each equation, let x represent the number of games played and let y represent the total cost.

$y = 2.50x + 2$

$y = 2x + 4$

b. Graph the system. What is the solution and what does it represent?



The solution is (4, 12). The cost at both alleys will be the same for 4 games bowled; that cost will be $12.

8. Multi-Step Jeremy runs 7 miles per week and increases his distance by 1 mile each week. Tony runs 3 miles per week and increases his distance by 2 miles each week. In how many weeks will Jeremy and Tony be running the same distance? What will that distance be?

4 weeks; 11 miles

9. Critical Thinking Write a real-world situation that could be represented by the system of equations shown below.

$$\begin{cases} y = 4x + 10 \\ y = 3x + 15 \end{cases}$$

Sample answer: Store A rents carpet cleaners for a fee of $10, plus $4 per day. Store B rents carpet cleaners for a fee of $15, plus $3 per day.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

10. Multistep The table shows two options provided by a high-speed Internet provider.

	Setup Fee ($)	Cost per Month ($)
Option 1	50	30
Option 2	No setup fee	$40

a. In how many months will the total cost of both options be the same? What will that cost be?

5 months; $200

b. If you plan to cancel your Internet service after 9 months, which is the cheaper option? Explain.

Option 1 is cheaper

Option 1: cost $= 30(9) + 50 = \$320$

Option 2: cost $= 40(9) = \$360$

11. Draw Conclusions How many solutions does the system formed by $x - y = 3$ and $ay - ax + 3a = 0$ have for a nonzero number a? Explain.

Infinitely many; sample answer: Rearranging the left side of the 2nd equation and subtracting $3a$ from both sides gives $-ax + ay = -3a$. Dividing both sides by $-a$ gives $x - y = 3$. So, the equations describe the same line.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Have students imagine adding a third equation with the same variables to a system of two linear equations in two variables.

What is the solution to a system of three equations in two variables? The ordered pair that solves all 3 equations.

How can you write a third equation to include in a system of two linear equations in two variables and be sure you won't change the solution of the system? Use the technique for writing an equation of a line through two known points, and use the point that is the solution of the original system as one of the points.

Lesson Support

Content Objective Students will learn how to solve a system of linear equations by substitution.

Language Objective Students will describe the steps for how to solve a system of linear equations by substitution.

California Common Core Standards

CA CC **8.EE.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

CA CC **8.EE.8c** Solve real-world and mathematical problems leading to two linear equations in two variables.

CA CC **MP.6** Attend to precision.

CA CC **Focus | Coherence | Rigor**

Building Background

Connecting to Everyday Life Discuss situations in which substitutions occur in everyday life. For example, there may be a substitute teacher if a teacher is absent, or one player may be substituted for another if a player is injured in a game. Then ask students to describe situations in which they use substitution in math. For example, a solution may be substituted into an original equation to check an answer, or a value may be substituted into an expression to evaluate the expression.

$$\begin{aligned} 3x + 5 &= -1 \\ x &\stackrel{?}{=} -2 \\ 3(-2) + 5 &\stackrel{?}{=} -1 \\ -6 + 5 &\stackrel{?}{=} -1 \\ -1 &= -1 \end{aligned}$$

Learning Progressions

In this lesson, students solve a system of two linear equations by the substitution method. Important understandings for students include the following:

- Solve a system of linear equations algebraically using substitution.
- Use a graph to estimate the solution of a system before solving algebraically.
- Solve problems with systems of linear equations algebraically.

In this lesson students build on the graphical foundation they set in the last lesson for solving a system. They now begin to solve a system algebraically by solving one equation for a variable and substituting it into the other equation. They refer back to the graphs of systems to both check and estimate solutions. They continue to solve problems by applying the new methodology to create and solve a system based on real-life situations.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Analyze and solve linear equations and pairs of simultaneous linear equations.*** Have students consider the following the following system:

$$\begin{cases} \frac{y}{2} - x = 1 \\ x + y + 7 = 0 \end{cases}$$

Explain the following method for solving the system, which is similar to the substitution method: (1) Solve both equations for the same variable. (2) Set the two expressions for that same variable equal to each other, and solve for the variable. (3) Solve for the remaining variable. Ask students to use the method to solve the system.

Sample answer: Solve each equation for x: $x = \frac{y}{2} - 1$ and $x = -y - 7$. Then $\frac{y}{2} - 1 = -y - 7$ and $y = -4$. Substituting, $x = -3$. The solution is $(-4, -3)$.

Language Support

California ELD Standards

CA ELD **Emerging 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering basic questions with prompting and substantial support.

CA ELD **Expanding 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering detailed questions with occasional prompting and moderate support.

CA ELD **Bridging 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering detailed questions with minimal prompting and support.

Linguistic Support

Academic/Content Vocabulary

Provide scaffolding for English learners by walking them step-by-step through Example 1 as they rephrase each step in their own words.

Remind students that the new vocabulary introduced in the lesson is highlighted in yellow, which means that it can be found in the glossary.

Background Knowledge

Students have likely had a class with a substitute teacher who takes the place of the regular classroom teacher. The idea of a substitution is also familiar in sports like soccer or basketball or in cooking, where you can substitute one ingredient for another. Help students understand the concept of the substitution method by making a connection to these situations.

Leveled Strategies for English Learners

Emerging Have students work in mixed proficiency level teams to solve the *Treasure Island* problem in Example 3. Have students predict by writing out a solution they think will find the treasure.

Expanding As students work together through the *Pirate Treasure* problem in Example 3, have students tell how they are solving each step. Have a partner write down the steps to share with the class when finished.

Bridging Using Example 3, have students work with a small team to predict the solution, solve the problem, and explain each step of the solution. Have students name the island if they solve the problem correctly.

Math Talk To help students answer the question posed in Math Talk, provide a sentence frame for students to use to respond.

I know that (—5, —2) is not the solution because _____.

LESSON

8.2 Solving Systems by Substitution

CA Common Core Standards

The student is expected to:

CA CC Expressions and Equations—8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

CA CC Expressions and Equations—8.EE.8c

Solve real-world and mathematical problems leading to two linear equations in two variables.

Mathematical Practices

CA CC MP.6 Precision

Engage

ESSENTIAL QUESTION

How do you use substitution to solve a system of linear equations? Sample answer: Solve for one variable in one of the equations. Substitute the resulting expression for the same variable in the other equation. Substitute the solution into either original equation to find the value of the other variable.

Motivate the Lesson

Ask: How can you turn two equations with two variables into one equation with one variable? Begin the Lesson to find out.

Explore

Lead students in a discussion of the **Substitution Property.** For example, because $5 = 3 + 2$, $3 + 2$ can be substituted for 5 in any expression and 5 can be substituted for $3 + 2$. Note that this also works for variable expressions: if $y = x + 5$, then $x + 5$ can be substituted for y. The **substitution method** for solving a linear system of equations makes use of the Substitution Property.

Explain

EXAMPLE 1

Questioning Strategies **Mathematical Practices**

- Why not solve for x in Step 1? You can solve for x in Step 1, but then you will have fractions in Step 2 of the solution.
- Is (1, 4) the only solution to this system? The graph in Step 4 shows that the graphs of the equations intersect at only one point, so there is only one solution.

Focus on Critical Thinking **Mathematical Practices**

Make sure students understand that in Step 1 either equation could be used and either variable could be solved for. However, it is usually easier to solve for a variable that has a coefficient of 1.

YOUR TURN

Avoid Common Errors

Some students may have difficulty deciding which equation to use in Step 1 of the solution process. Discuss what should be considered when choosing an equation to solve for a variable. For example, in Exercise 4, the most straightforward method would be to solve either equation for y, thereby avoiding fractions in the solution process.

ADDITIONAL EXAMPLE 1

Solve the system of linear equations by substitution. Check your answer.

$$\begin{cases} 2x + y = 5 \\ -3x + 2y = 17 \end{cases}$$

(–1, 7)

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LESSON 8.2

Solving Systems by Substitution

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

ESSENTIAL QUESTION How do you use substitution to solve a system of linear equations?

Solving a Linear System by Substitution

The **substitution method** is used to solve systems of linear equations by solving an equation for one variable and then substituting the resulting expression for that variable into the other equation. The steps for this method are as follows:

1. Solve one of the equations for one of its variables.
2. Substitute the expression from step 1 into the other equation and solve for the other variable.
3. Substitute the value from step 2 into either original equation and solve to find the value of the variable in step 1.

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EXAMPLE 1

CA CC 8.EE.8b

Solve the system of linear equations by substitution. Check your answer.

$$\begin{cases} -3x + y = 1 \\ 4x + y = 8 \end{cases}$$

STEP 1 Solve an equation for one variable.

$-3x + y = 1$ *Select one of the equations.*

$y = 3x + 1$ *Solve for the variable y. Isolate y on one side.*

STEP 2 Substitute the expression for y in the other equation and solve.

$4x + (3x + 1) = 8$ *Substitute the expression for the variable y.*

$7x + 1 = 8$ *Combine like terms.*

$7x = 7$ *Subtract 1 from each side.*

$x = 1$ *Divide each side by 7.*

STEP 3 Substitute the value of x you found into one of the equations and solve for the other variable, y.

$-3(1) + y = 1$ *Substitute the value of x into the first equation.*

$-3 + y = 1$ *Simplify.*

$y = 4$ *Add 3 to each side.*

So, (1, 4) is the solution of the system.

My Notes

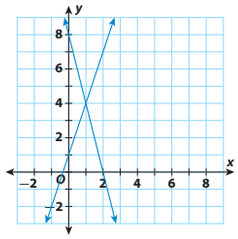
My Notes

STEP 4 Check the solution by graphing.

$-3x + y = 1$ $4x + y = 8$

x-intercept: $-\frac{1}{3}$ x-intercept: 2

y-intercept: 1 y-intercept: 8



The point of intersection is (1, 4).

Reflect

1. **Justify Reasoning** Is it more efficient to solve $-3x + y = 1$ for x? Why or why not?
   No; more steps are needed, and $x = \frac{1}{3}y - \frac{1}{3}$ is more difficult to substitute.
2. Is there another way to solve the system?
   Yes; solve $4x + y = 8$ for y and substitute that expression into $-3x + y = 1$.
3. What is another way to check your solution?
   Substitute (1, 4) into each equation and see if both equations are true.

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YOUR TURN

Solve each system of linear equations by substitution.

4. $\begin{cases} 3x + y = 11 \\ -2x + y = 1 \end{cases}$ (2, 5)

5. $\begin{cases} 2x - 3y = -24 \\ x + 6y = 18 \end{cases}$ (−6, 4)

6. $\begin{cases} x - 2y = 5 \\ 3x - 5y = 8 \end{cases}$ (−9, −7)

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.6

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to attend to precision. Students examine graphs of systems of equations to understand why the graphical method of solving a system is not always able to provide a precise solution. They learn that a graph of a system of equations can provide an estimate of the coordinates of the solution. Students also learn to use the algebraic method of substitution to find the precise solution to a system of equations.

Math Background

The systems of equations in this lesson are linear equations with one solution. The graphs of these equations are lines that intersect at one point with the coordinates given by the solution of the system. Some systems of linear equations, however, have no solution, and some have an infinite number of solutions. When the equations graphed are parallel lines, there is no solution to the system. Algebraically the result will be a false statement, such as $3 = -2$. When the equations graphed are the same line, then there are an infinite number of solutions. Algebraically the result will be a true statement, such as $0 = 0$.

ADDITIONAL EXAMPLE 2

Solve the system $\begin{cases} 2x + 6y = -1 \\ 6x - 21y = 10 \end{cases}$.

$\left(\frac{1}{2}, \frac{1}{3}\right)$

Interactive Whiteboard
Interactive example available online

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EXAMPLE 2

Questioning Strategies Mathematical Practices

- How could you change the graph to get closer to the exact coordinates of the intersection? If the scales of the axes are changed to smaller increments, the estimated coordinates of the point of intersection will be closer to their exact values.
- How can you check that $\left(-\frac{24}{5}, -\frac{11}{5}\right)$ is the exact solution? Substitute the values in for x and y in both original equations. If this is the exact solution, both will result in a true statement.

Focus on Technology Mathematical Practices

Suggest students use a graphing calculator to graph both equations in the same window and use the TRACE function on the calculator to find the approximate coordinates of the intersection. They could also use the 2nd CALC **Intersect** function to have their calculator show the coordinates of the intersection.

YOUR TURN

Engage with the Whiteboard

Have a volunteer graph each of the lines and estimate the intersection. Then have another student find the algebraic solution and compare it to the estimate.

ADDITIONAL EXAMPLE 3

A line passes through the points (5, 3) and (−5, −2). Another line passes through the points (−6, 4) and (2, −4). Find the coordinates of the intersection of the two lines.

$\left(-\frac{5}{3}, -\frac{1}{3}\right)$

Interactive Whiteboard
Interactive example available online

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EXAMPLE 3

Questioning Strategies Mathematical Practices

- Will the x- and y-coordinates of the intersection of the lines be positive or negative? The intersection is in the second quadrant, so the x-coordinate is negative and the y-coordinate is positive.
- How does writing both equations in slope-intercept form help you solve for x? When both equations are in $y = mx + b$ format, the expressions for $mx + b$ in each can be set equal to each other and then solved for x.

Focus on Critical Thinking

Explain that any two points on a line can be used to find the slope, but the coordinates of the points need to be exact. In this situation the coordinates of the points A, B, C, and D can be found exactly.

Using a Graph to Estimate the Solution of a System

You can use a graph to estimate the solution of a system of equations before solving the system algebraically.

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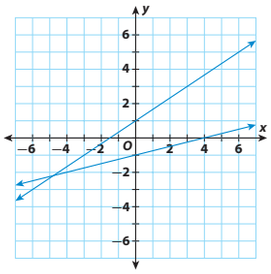
EXAMPLE 2

CA CC 8.EE.8b

Solve the system $\begin{cases} x - 4y = 4 \\ 2x - 3y = -3 \end{cases}$.

STEP 1 Sketch a graph of each equation by substituting values for x and generating values of y.



STEP 2 Find the intersection of the lines. The lines appear to intersect near $(-5, -2)$.

STEP 3 Solve the system algebraically.

Solve $x - 4y = 4$ for x.

$x - 4y = 4$
$x = 4 + 4y$

Substitute to find y.

$2(4 + 4y) - 3y = -3$
$8 + 8y - 3y = -3$
$8 + 5y = -3$
$5y = -11$
$y = -\frac{11}{5}$

Substitute to find x.

$x = 4 + 4y$
$= 4 + 4\left(-\frac{11}{5}\right)$
$= \frac{20 - 44}{5}$
$= -\frac{24}{5}$

The solution is $\left(-\frac{24}{5}, -\frac{11}{5}\right)$.

Math Talk Mathematical Practices
In Step 2, how can you tell that $(-5, -2)$ is not the solution?

The lines intersect near, but not at, $(-5, -2)$. Also, if you substitute $x = -5$ and $y = -2$ into each equation, you get false statements.

STEP 4 Use the estimate you made using the graph to judge the reasonableness of your solution.

$-\frac{24}{5}$ is close to the estimate of -5, and $-\frac{11}{5}$ is close to the estimate of -2, so the solution seems reasonable.

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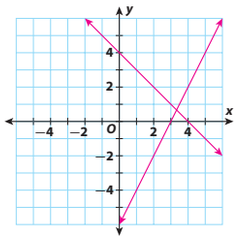
YOUR TURN

7. Estimate the solution of the system $\begin{cases} x + y = 4 \\ 2x - y = 6 \end{cases}$ by sketching a graph of each linear function. Then solve the system algebraically. Use your estimate to judge the reasonableness of your solution.



The estimated solution is (3, 1).

The algebraic solution is $\left(\frac{10}{3}, \frac{2}{3}\right)$.

The solution **is**/is not reasonable because $\frac{10}{3}$ is close to the estimate of 3, and $\frac{2}{3}$ is close to the estimate of 1.

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Solving Problems with Systems of Equations

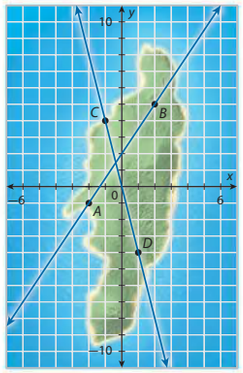
EXAMPLE 3

Real World

CA CC 8.EE.8c

As part of Class Day, the eighth grade is doing a treasure hunt. Each team is given the following riddle and map. At what point is the treasure located?

There's pirate treasure to be found. So search on the island, all around. Draw a line through A and B. Then another through C and D. Dance a jig, "X" marks the spot. Where the lines intersect, that's the treasure's plot!



STEP 1 Give the coordinates of each point and find the slope of the line through each pair of points.

A: $(-2, -1)$ C: $(-1, 4)$

B: $(2, 5)$ D: $(1, -4)$

Slope:
$\frac{5 - (-1)}{2 - (-2)} = \frac{6}{4}$
$= \frac{3}{2}$

Slope:
$\frac{-4 - 4}{1 - (-1)} = \frac{-8}{2}$
$= -4$

$\left(-\frac{1}{3}, \frac{3}{2}\right)$: This ordered pair can be used as an estimate for the solution.

Math Talk Mathematical Practices
Where do the lines appear to intersect? How is this related to the solution?

DIFFERENTIATE INSTRUCTION

Critical Thinking

Give students two or more systems of equation, such as those below. Have them complete tables that show ordered pair solutions for each of the equations, for all integers from $x = -5$ to $x = 5$. Ask them to find the solution to each system by identifying the ordered pair that appears in both tables for that system.

$\begin{cases} 2x + y = 7 \\ x - 3y = 0 \end{cases}$ $\begin{cases} 2x - 5y = 1 \\ -3x + 4y = 2 \end{cases}$

(3, 1) $(-2, -1)$

Multiple Representations

One way to use substitution to solve a system of equations is to rewrite both equations so that the same variable is isolated and set equal to an expression. Then the Transitive Property can be used to set the expressions equal to each other.

Rewriting the equations in the system found in Example 1 results in $y = 3x + 1$ and $y = -4x + 8$. Using the Transitive Property:

$3x + 1 = -4x + 8$
$7x = 7$ and so $x = 1$

Substitute 1 for x in one of the original equations to find the value of y.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

YOUR TURN

Engage with the Whiteboard

Have volunteers write the two equations that represent the car rentals. Ask another volunteer to demonstrate how to solve the system.

Talk About It

Check for Understanding

Ask: Ronald says that (20, 0.25) is the only combination of cost per day and cost per mile that could have given Carlos his $120 total. Is he right? Justify your answer. No, Ronald is wrong. Any point on the graph of the line that represents Carlos's situation would be a solution. However, (20, 0.25) is the only combination that would work for both Carlos's and Vanessa's situation.

Elaborate

Talk About It

Summarize the Lesson

Ask: How can a graph of a system of two equations help you determine if your algebraic solution is reasonable? You can see which quadrant the intersection is in and the approximate value of the intersection by looking at the graph. You can compare the algebraic solution to these two pieces of information and confirm that it is reasonable.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 5–8, have volunteers graph the equations on the whiteboard and estimate the solution.

Avoid Common Errors

Exercises 5–8 Remind students that to estimate the solution of each system they will first need to graph each system.

Exercise 9 Remind students to first analyze their two equations to determine which has a variable with a coefficient of 1. Then caution students to use the Distributive Property when substituting. So when $y = -3x + 163$ and $2x + 3(-3x + 163) = 174$, the 3 must be distributed to both $-3x$ and 163.

STEP 2 Write equations in slope-intercept form describing the line through points A and B and the line through points C and D.

Line through A and B:

Use the slope and a point to find b.

$5 = \left(\frac{3}{2}\right)2 + b$
$b = 2$

The equation is $y = \frac{3}{2}x + 2$.

Line through C and D:

Use the slope and a point to find b.

$4 = -4(-1) + b$
$b = 0$

The equation is $y = -4x$.

STEP 3 Solve the system algebraically.

Substitute $\frac{3}{2}x + 2$ for y in $y = -4x$ to find x.

$\frac{3}{2}x + 2 = -4x$
$\frac{11}{2}x = -2$
$x = -\frac{4}{11}$

Substitute to find x.
$y = -4\left(-\frac{4}{11}\right) = \frac{16}{11}$

The solution is $\left(-\frac{4}{11}, \frac{16}{11}\right)$.

My Notes

YOUR TURN

8. Ace Car Rental rents cars for x dollars per day plus y dollars for each mile driven. Carlos rented a car for 4 days, drove it 160 miles, and spent \$120. Vanessa rented a car for 1 day, drove it 240 miles, and spent \$80. Write equations to represent Carlos's expenses and Vanessa's expenses. Then solve the system and tell what each number represents.

Carlos: $4x + 160y = 120$

Vanessa: $x + 240y = 80$

(20, 0.25);

20 represents the cost per day: \$20;

0.25 represents the cost per mile: \$0.25

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Guided Practice

Solve each system of linear equations by substitution. (Example 1)

1. $\begin{cases} 3x - 2y = 9 \\ y = 2x - 7 \end{cases}$ (5, 3)

2. $\begin{cases} y = x - 4 \\ 2x + y = 5 \end{cases}$ (3, −1)

3. $\begin{cases} x + 4y = 6 \\ y = -x + 3 \end{cases}$ (2, 1)

4. $\begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$ (4, 1)

Solve each system. Estimate the solution first. (Example 2)

5. $\begin{cases} 6x + y = 4 \\ x - 4y = 19 \end{cases}$

Estimate (1, −4)

Solution $\left(\frac{7}{5}, -\frac{22}{5}\right)$

6. $\begin{cases} x + 2y = 8 \\ 3x + 2y = 6 \end{cases}$

Estimate (−1, 5)

Solution $\left(-1, \frac{9}{2}\right)$

7. $\begin{cases} 3x + y = 4 \\ 5x - y = 22 \end{cases}$

Estimate (3, −6)

Solution $\left(\frac{13}{4}, -\frac{23}{4}\right)$

8. $\begin{cases} 2x + 7y = 2 \\ x + y = -1 \end{cases}$

Estimate (−2, 1)

Solution $\left(-\frac{9}{5}, \frac{4}{5}\right)$

9. Adult tickets to Space City amusement park cost x dollars. Children's tickets cost y dollars. The Henson family bought 3 adult and 1 child tickets for \$163. The Garcia family bought 2 adult and 3 child tickets for \$174. (Example 3)

a. Write equations to represent the Hensons' cost and the Garcias' cost.

Hensons' cost: $3x + y = 163$ Garcias' cost: $2x + 3y = 174$

b. Solve the system.

adult ticket price: \$45 child ticket price: \$28

ESSENTIAL QUESTION CHECK-IN

10. How can you decide which variable to solve for first when you are solving a linear system by substitution?

Choose the variable whose coefficient is 1. If no coefficient is 1, choose the variable with the least positive integer coefficient.

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8.2 LESSON QUIZ

CA CC **8.EE.8b, 8.EE.8c**

Solve the system of linear equations by substitution. Check your answer.

1. $\begin{cases} x + y = 5 \\ 2x - y = 7 \end{cases}$

2. $\begin{cases} y = -2x + 6 \\ -4x - 6y = 4 \end{cases}$

Solve each system. Estimate the solution first.

3. $\begin{cases} 3x + y = 7 \\ -7x - 5y = 25 \end{cases}$

4. $\begin{cases} x + 3y = 9 \\ 2x + 4y = 7 \end{cases}$

5. Jill bought oranges and bananas. She bought 12 pieces of fruit and spent \$5. Oranges cost \$0.50 each and bananas cost \$0.25 each. Write a system of equations to model the problem. Then solve the system algebraically. How many oranges and how many bananas did Jill buy?

Lesson Quiz available online

my.hrw.com

Answers

1. (4, 1)

2. (5, −4)

3. $\left(\frac{15}{2}, -\frac{31}{2}\right)$

4. $\left(-\frac{15}{2}, \frac{11}{2}\right)$

5. $x + y = 12$
$0.50x + 0.25y = 5.00$
(8, 4); 8 oranges and 4 bananas

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.EE.8b, 8.EE.8c**

Concepts & Skills	Practice
Example 1 Solving a Linear System by Substitution	Exercises 1–4, 14
Example 2 Using a Graph to Estimate the Solution of a System	Exercises 5–8, 11
Example 3 Solving Problems with Systems of Equations	Exercises 9, 12–13, 15

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
11	**3** Strategic Thinking H.O.T.	**MP.3** Logic
12–13	**2** Skills/Concepts	**MP.4** Modeling
14	**2** Skills/Concepts	**MP.1** Problem Solving
15	**2** Skills/Concepts	**MP.4** Modeling
16	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure
17	**3** Strategic Thinking H.O.T.	**MP.6** Precision
18	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

8.2 Independent Practice

CACC 8.EE.8b, 8.EE.8c

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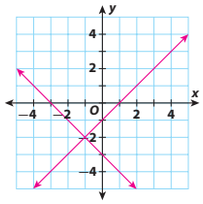
11. Check for Reasonableness Zach solves the system $\begin{cases} x + y = -3 \\ x - y = 1 \end{cases}$ and finds the solution $(1, -2)$. Use a graph to explain whether Zach's solution is reasonable.

The graph shows that the x-coordinate of the solution is negative, so Zach's solution is not reasonable.



12. Represent Real-World Problems Angelo bought apples and bananas at the fruit stand. He bought 20 pieces of fruit and spent \$11.50. Apples cost \$0.50 and bananas cost \$0.75 each.

a. Write a system of equations to model the problem. (Hint: One equation will represent the number of pieces of fruit. A second equation will represent the money spent on the fruit.)

$\begin{cases} x + y = 20 \\ 0.50x + 0.75y = 11.50 \end{cases}$

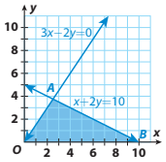
b. Solve the system algebraically. Tell how many apples and bananas Angelo bought.

14 apples and 6 bananas

13. Represent Real-World Problems A jar contains n nickels and d dimes. There is a total of 200 coins in the jar. The value of the coins is \$14.00. How many nickels and how many dimes are in the jar?

120 nickels and 80 dimes

14. Multistep The graph shows a triangle formed by the x-axis, the line $3x - 2y = 0$, and the line $x + 2y = 10$. Follow these steps to find the area of the triangle.



a. Find the coordinates of point A by solving the system $\begin{cases} 3x - 2y = 0 \\ x + 2y = 10 \end{cases}$.

Point A: $\left(\frac{5}{2}, \frac{15}{4}\right)$

b. Use the coordinates of point A to find the height of the triangle.

height: $\frac{15}{4}$ units

c. What is the length of the base of the triangle?

base: 10 units

d. What is the area of the triangle? $18\frac{3}{4}$ square units

15. Jed is graphing the design for a kite on a coordinate grid. The four vertices of the kite are at $A\left(-\frac{4}{3}, \frac{2}{3}\right)$, $B\left(\frac{14}{3}, -\frac{4}{3}\right)$, $C\left(\frac{14}{3}, -\frac{16}{3}\right)$, and $D\left(\frac{2}{3}, -\frac{16}{3}\right)$. One kite strut will connect points A and C. The other will connect points B and D. Find the point where the struts cross.

$\left(\frac{8}{3}, -\frac{10}{3}\right)$

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

16. Analyze Relationships Consider the system $\begin{cases} 6x - 3y = 15 \\ x + 3y = -8 \end{cases}$. Describe three different substitution methods that can be used to solve this system. Then solve the system.

Solve the second equation for x ($x = -8 - 3y$) and then substitute that value into the first equation. Solve the first equation for y ($y = 2x - 5$) and then substitute that value into the second equation. Solve either equation for $3y$ ($3y = -8 - x$ or $3y = 6x - 15$) and then substitute that value into the other equation. Solution: $(1, -3)$

17. Communicate Mathematical Ideas Explain the advantages, if any, that solving a system of linear equations by substitution has over solving the same system by graphing.

The substitution method has the advantage of always giving an exact answer. Graphing produces an exact answer only if the solution is an ordered pair whose coordinates are integers.

18. Persevere in Problem Solving Create a system of equations of the form $\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$ that has $(7, -2)$ as its solution. Explain how you found the system.

Sample answer: $\begin{cases} x + 3y = 1 \\ -2x - 4y = -6 \end{cases}$; I chose random values of A, B, D, and E, substituted them into the equations, and calculated the values of C and F using $x = 7$ and $y = -2$.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Let m represent a real number. Solve the following system of equations. The solution should be in the form (x, y), and x and y will be expressed in terms of m.

$\begin{cases} 8x + y = m \\ 4x + 3y = m \end{cases}$

$\left(\frac{1}{10}m, \frac{2}{10}m\right)$

Lesson Support

Content Objective Students will learn how to solve a system of linear equations by adding or subtracting.

Language Objective Students will explain how to solve a system of linear equations by adding or subtracting.

California Common Core Standards

CA CC **8.EE.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

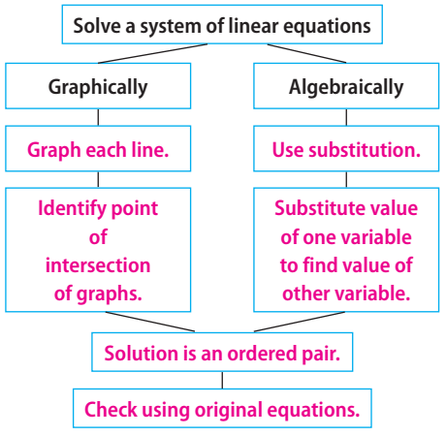
CA CC **8.EE.8c** Solve real-world and mathematical problems leading to two linear equations in two variables.

CA CC **MP.1** Make sense of problems and persevere in solving them.

CA CC **Focus | Coherence | Rigor**

Building Background

Eliciting Prior Knowledge Have students create a main idea web to summarize what they have learned about solving systems of linear equations. They should include and describe the graphical method and the algebraic method of substitution. Sample completed web is shown.



Learning Progressions

In this lesson, students solve a system of two linear equations using the elimination method. Important understandings for students include the following:

- Solve a linear system by adding.
- Solve a linear system by subtracting.
- Solve problems with systems of linear equations by elimination.

Students continue to solve systems of equations algebraically. They learn the forms to look for to decide when the elimination method is appropriate. They recognize that there are a variety of approaches to solving a system and that with experience they will be increasingly able to select the easiest method. They continue to solve real-life problems that illustrate the importance of being able to formulate and solve a system of linear equations.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Analyze and solve linear equations and pairs of simultaneous linear equations.*** Remind students that they know how to solve equations with rational coefficients and constants. Ask them to apply what they have learned about elimination to solve the following system:

$$\begin{cases} \frac{3}{2}x - \frac{1}{3}y = 14 \\ \frac{5}{4}x + \frac{1}{3}y = 12 \end{cases}$$

(8, –6)

Language Support

California ELD Standards

CA ELD **Emerging 2.I.1.** **Exchanging information/ideas** – Contribute to conversations and express ideas by asking and answering yes-no and wh-questions and responding using short phrases.

CA ELD **Expanding 2.I.1.** **Exchanging information/ideas** – Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, asking relevant questions, affirming others, and adding relevant information.

CA ELD **Bridging 2.I.1.** **Exchanging information/ideas** – Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, adding relevant information, building on responses, and providing useful feedback.

Linguistic Support

Academic/Content Vocabulary

Provide scaffolding for English learners by walking them step-by-step through Example 1 as they rephrase each step in their own words.

Remind students that the new vocabulary introduced in the lesson is highlighted in yellow, which means that it can be found in the glossary. The word *elimination* is a cognate in Spanish.

Idioms and Expressions

Students may hear or read the expression *process of elimination*. A *process of elimination* involves many tests to eliminate suspects one-at-a-time until only one possibility remains. In sports tournaments, the winner is determined using a *process of elimination*. In this lesson, students learn the *elimination* method to solve a system of linear equations. Much like with the process of elimination, solving a system by elimination involves eliminating, or *getting rid of*, one of the variables so that only one remains.

Leveled Strategies for English Learners

Emerging Have students work together to solve Exercise 7. Have them show each step to demonstrate how to decide whether to add or subtract to eliminate the variable.

Expanding Have students think of when the method of elimination is the preferred method for solving a system of linear equations.

Bridging Have students work with a partner to select a problem from this lesson, solve it using the elimination method, and tell step-by-step how to solve it.

Math Talk Explain to English learners that the question is not simply providing two possible methods for them to choose from, but they need to tell why they choose the method that they do. Provide a sentence frame.

It is better to check a solution by _____ because _____.

LESSON

8.3 Solving Systems by Elimination

CA Common Core Standards

The student is expected to:

CA CC Expressions and Equations—8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

CA CC Expressions and Equations—8.EE.8c

Solve real-world and mathematical problems leading to two linear equations in two variables.

Mathematical Practices

CA CC MP.1 Problem Solving

Engage

ESSENTIAL QUESTION

How do you solve a system of linear equations by adding or subtracting? Sample answer: Write the equations so that like terms are aligned vertically. Add or subtract the equations to eliminate either the x or y variable. Simplify to solve for the variable that was not eliminated. Then, substitute that value into one of the original equations to solve for the other variable.

Motivate the Lesson

Ask: How can you turn two equations with two variables into one equation with one variable without substituting? Begin the lesson to find out.

Explore

Remind students of the Addition and Subtraction Properties of Equality, where the same quantity can be added or subtracted from both sides of an equation. Ask them to think about adding or subtracting two equations. Have them discuss whether the properties of equality apply in this case.

Explain

EXAMPLE 1

Questioning Strategies **Mathematical Practices**

- What type of equations would have both the x and y values eliminated when combining the two equations? The x and y terms would have to both be the same or both be opposites. The constant terms could be different.
- In Step 2, if 6 is substituted for x in the other equation, $2x - 3y = 12$, what is the value of y? $y = 0$

Focus on Critical Thinking

Make sure students understand that the addition shown in Step 1 is a way to combine the information given in both equations into one equation with one variable.

YOUR TURN

Avoid Common Errors

In Exercise 5, be sure students correctly align the terms in the equations. The constant terms should be aligned by place value, so that $4 + 20$ is correctly found to be 24, not 60.

ADDITIONAL EXAMPLE 1

Solve the system of equations by adding. Check your answer.

$$\begin{cases} 2x + y = 8 \\ -2x + 3y = 16 \end{cases}$$

(1, 6)

Interactive Whiteboard

Interactive example available online

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LESSON 8.3

Solving Systems by Elimination

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

ESSENTIAL QUESTION

How do you solve a system of linear equations by adding or subtracting?

Solving a Linear System by Adding

The **elimination method** is another method used to solve a system of linear equations. In this method, one variable is *eliminated* by adding or subtracting the two equations of the system to obtain a single equation in one variable. The steps for this method are as follows:

1. Add or subtract the equations to eliminate one variable.
2. Solve the resulting equation for the other variable.
3. Substitute the value into either original equation to find the value of the eliminated variable.

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EXAMPLE 1

CA CC 8.EE.8b

Solve the system of equations by adding. Check your answer.

$$\begin{cases} 2x - 3y = 12 \\ x + 3y = 6 \end{cases}$$

STEP 1 Add the equations.

$2x - 3y = 12$	Write the equations so that like terms are aligned.
$+ (x + 3y = 6)$	Notice that the terms $-3y$ and $3y$ are opposites.
$3x + 0 = 18$	Add to eliminate the variable y.
$3x = 18$	Simplify and solve for x.
$\frac{3x}{3} = \frac{18}{3}$	Divide each side by 3.
$x = 6$	Simplify.

STEP 2 Substitute the solution into one of the original equations and solve for y.

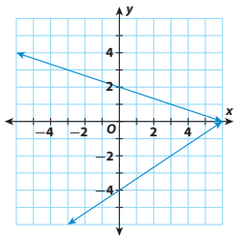
$x + 3y = 6$	Use the second equation.
$6 + 3y = 6$	Substitute 6 for the variable x.
$3y = 0$	Subtract 6 from each side.
$y = 0$	Divide each side by 3 and simplify.

My Notes

STEP 3 Write the solution as an ordered pair: (6, 0)

STEP 4 Check the solution by graphing.

$2x - 3y = 12$	$x + 3y = 6$
x-intercept: 6	x-intercept: 6
y-intercept: -4	y-intercept: 2



The point of intersection is (6, 0).

Substituting; if the x- and y-values of the solution are not integers, graphing will not produce an accurate check.

Math Talk Mathematical Practices

Is it better to check a solution by graphing or by substituting the values in the original equations?

Reflect

1. Can this linear system be solved by subtracting one of the original equations from the other? Why or why not?
   No; if either of the original equations is subtracted from the other, neither variable will be eliminated.
2. What is another way to check your solution?
   Substitute (6, 0) into each equation and see if both equations are true.

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YOUR TURN

Solve each system of equations by adding. Check your answers.

3. $\begin{cases} x + y = -1 \\ x - y = 7 \end{cases}$ (3, −4)
4. $\begin{cases} 2x + 2y = -2 \\ 3x - 2y = 12 \end{cases}$ (2, −3)
5. $\begin{cases} 6x + 5y = 4 \\ -6x + 7y = 20 \end{cases}$ (−1, 2)

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.1

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to make sense of problems and persevere in solving them. Students solve systems of equations using either addition or subtraction to eliminate one of the variables. Then students ask themselves if the solution they found makes sense. They use what they learned about graphing systems of equations to check the accuracy or reasonableness of the solution they found. They also learn to translate real-world problems into systems of equations and solve them.

Math Background

Systems of equations containing more than two equations and/or more than two variables can be solved by representing the equations with matrices. A matrix is a rectangular array of numbers. Matrix methods involve operations performed on the rows of numbers to create 0s in particular areas of the matrix. In part, the matrix method uses elimination. For example, the matrix for a simple two-equation system $\begin{cases} 3x + 5y = -2 \\ 3x - 5y = 8 \end{cases}$ is $\begin{bmatrix} 3 & 5 & -2 \\ 3 & -5 & 8 \end{bmatrix}$. After adding row 2 to row 1, the matrix becomes $\begin{bmatrix} 6 & 0 & 6 \\ 3 & -5 & 8 \end{bmatrix}$. From this matrix you can see that $6x = 6$, so $x = 1$. Using substitution, $y = -1$.

ADDITIONAL EXAMPLE 2

Solve the system of equations by adding. Check your answer.

$$\begin{cases} 2x + y = 8 \\ 2x + 3y = 4 \end{cases}$$

$(5, -2)$

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EXAMPLE 2

Questioning Strategies Mathematical Practices

- How can you tell when you should subtract rather than add the equations? One of the variable terms will be identical in both equations.
- In Step 2, if 3 is substituted for y in the other equation, $3x + 3y = 6$, what is the value of x? $x = -1$

Focus on Critical Thinking

Discuss with students how subtracting the second equation is the same as adding the opposite of each term, including those terms on the right side of the equal sign.

YOUR TURN

Engage with the Whiteboard

For Exercises 8–10, have volunteers write the subtraction sign in front of the second equation in each and then demonstrate how to subtract each term from the equation on top. Close attention should be paid to subtracting negative terms. Have other volunteers demonstrate subtracting the top equation from the bottom equation.

ADDITIONAL EXAMPLE 3

Jeb and Lori went to a florist to buy flowers. Jeb bought 6 roses and 3 carnations for \$20.25. Lori bought 8 roses and 3 carnations for \$25.75. Find the price of one rose and the price of one carnation.

rose: \$2.75; carnation: \$1.25

Interactive Whiteboard
Interactive example available online

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EXAMPLE 3

Questioning Strategies Mathematical Practices

- How does organizing the information in a table help you determine the equations to write for this situation? Would another column added to the table be helpful? If so, what would be included in that column? The table is helpful in that it clearly shows the cost for the shoes and stoves at both stores. A column for the amount spent at each store could be included to the right of the camp stoves column. With this column added to the table, the coefficients of the variables and the constant term of each equation are easily seen.
- How do you decide whether to add or subtract the two equations? The goal is to eliminate one of the variable terms. Since the coefficients of the variable y are the same in both equations, you subtract.
- Where should the club buy the equipment? They will save money buying at Top Sport, but since it is farther away, they may choose to save time and transportation costs by buying at Outdoor Explorer.

Focus on Communication

Emphasize the importance of naming the variables being used and what they represent. Any pair of variables can be used, but there must be one to represent the number of pairs of snowshoes and a different one to represent the number of camp stoves.

Solving a Linear System by Subtracting

If both equations contain the same x- or y-term, you can solve by subtracting.

EXAMPLE 2 CA CC 8.EE.8b

Solve the system of equations by subtracting. Check your answer.

$$\begin{cases} 3x + 3y = 6 \\ 3x - y = -6 \end{cases}$$

STEP 1 Subtract the equations.

$3x + 3y = 6$	Write the equations so that like terms are aligned.
$-(3x - y = -6)$	Notice that both equations contain the term $3x$.
$0 + 4y = 12$	Subtract to eliminate the variable x.
$4y = 12$	Simplify and solve for y.
$y = 3$	Divide each side by 4 and simplify.

STEP 2 Substitute the solution into one of the original equations and solve for x.

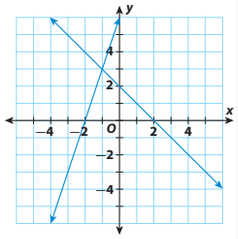
$3x - y = -6$	Use the second equation.
$3x - 3 = -6$	Substitute 3 for the variable y.
$3x = -3$	Add 3 to each side.
$x = -1$	Divide each side by 3 and simplify.

STEP 3 Write the solution as an ordered pair: $(-1, 3)$

STEP 4 Check the solution by graphing.

$3x + 3y = 6$	$3x - y = -6$
x-intercept: 2	x-intercept: -2
y-intercept: 2	y-intercept: 6

The point of intersection is $(-1, 3)$.



Reflect

6. **What If?** What would happen if you added the original equations?
You would get $6x + 2y = 0$. This does not help to solve the system. Neither variable would be eliminated.

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My Notes

7. How can you decide whether to add or subtract to eliminate a variable in a linear system? Explain your reasoning.
If the equations have two terms that are opposites, then you can add to eliminate a variable; if the equations have two terms that are the same, then you can subtract to eliminate a variable.

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YOUR TURN

Solve each system of equations by subtracting. Check your answers.

8. $\begin{cases} 6x - 3y = 6 \\ 6x + 8y = -16 \end{cases}$ $(0, -2)$

9. $\begin{cases} 4x + 3y = 19 \\ 6x + 3y = 33 \end{cases}$ $(7, -3)$

10. $\begin{cases} 2x + 6y = 17 \\ 2x - 10y = 9 \end{cases}$ $\left(7, \frac{1}{2}\right)$

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Solving Problems with Systems of Equations

Many real-world situations can be modeled and solved with a system of equations.

EXAMPLE 3 Real World CA CC 8.EE.8c

The Polar Bear Club wants to buy snowshoes and camp stoves. The club will spend $554.50 to buy them at Top Sports and $602.00 to buy them at Outdoor Explorer, before taxes, but Top Sports is farther away. How many of each item does the club intend to buy?

	Snowshoes	Camp Stoves
Top Sports	$79.50 per pair	$39.25
Outdoor Explorer	$89.00 per pair	$39.25

DIFFERENTIATE INSTRUCTION

Cooperative Learning

Divide students into groups of three and give each group a system of equations, such as the one below. Have one student solve the system by substitution, another solve by graphing, and the third solve by elimination. Have students discuss which method they think is the quickest and easiest way to solve their system.

$$\begin{cases} 2x + y = 7 \\ 8x - y = 3 \end{cases}$$

$(1, 5)$

Number Sense

Have students switch the order of the equations in Examples 1 and 2. Ask students if switching their order will affect the solution. Then have students solve the systems. Help students understand that switching the order does not affect the solution. Graphing the lines is a good way to establish this fact. The graphs (and the solution) remain the same regardless of the order in which the equations are graphed.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

YOUR TURN

Engage with the Whiteboard

Have a volunteer make a table to organize the information in the problem. Have another volunteer write the two equations for the system of equations. Ask another volunteer to demonstrate how to solve the system.

Elaborate

Talk About It

Summarize the Lesson

Ask: How do you know when to add or subtract when solving a system of equations by elimination? The goal is to eliminate one of the variable terms. If the coefficients of one variable term are the same in both equations, then you subtract. If they are opposites, you add.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 1, have volunteers explain the process of arriving at the correct value as they complete the write-in boxes for each step.

Avoid Common Errors

Exercise 3 Remind students to be cautious with the signs of the terms. Subtracting $-2y$ from y is the same as adding $2y$ to y. Suggest that if they have trouble correctly subtracting each term, they might choose to rewrite the second equation with the opposite of each term and then add the equations.

Exercise 8 Suggest that students organize the information in this situation in a table and identify what their variables represent.

STEP 1 Choose variables and write a system of equations.
Let x represent the number of pairs of snowshoes.
Let y represent the number of camp stoves.

Top Sports cost: $79.50x + 39.25y = 554.50$
Outdoor Explorer cost: $89.00x + 39.25y = 602.00$

STEP 2 Subtract the equations.

$79.50x + 39.25y = 554.50$	Both equations contain the term 39.25y.
$-(89.00x + 39.25y = 602.00)$	
$-9.50x + 0 = -47.50$	Subtract to eliminate the variable y.
$-9.50x = -47.50$	Simplify and solve for x.
$\frac{-9.50x}{-9.50} = \frac{-47.50}{-9.50}$	Divide each side by −9.50.
$x = 5$	Simplify.

STEP 3 Substitute the solution into one of the original equations and solve for y.

$79.50x + 39.25y = 554.50$	Use the first equation.
$79.50(5) + 39.25y = 554.50$	Substitute 5 for the variable x.
$397.50 + 39.25y = 554.50$	Multiply.
$39.25y = 157.00$	Subtract 397.50 from each side.
$\frac{39.25y}{39.25} = \frac{157.00}{39.25}$	Divide each side by 39.25.
$y = 4$	Simplify.

STEP 4 Write the solution as an ordered pair: (5, 4)

The club intends to buy 5 pairs of snowshoes and 4 camp stoves.

My Notes

YOUR TURN

11. At the county fair, the Baxter family bought 6 bags of roasted almonds and 4 juice drinks for $16.70. The Farley family bought 3 bags of roasted almonds and 4 juice drinks for $10.85. Find the price of a bag of roasted almonds and the price of a juice drink.

bag of roasted almonds: $1.95; juice drink: $1.25

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Guided Practice

1. Solve the system $\begin{cases} 4x + 3y = 1 \\ x - 3y = -11 \end{cases}$ by adding. (Example 1)

STEP 1 Add the equations.

$4x + 3y = 1$	Write the equations so that like terms are aligned.
$+\ \ x - 3y = -11$	
$5x + \boxed{0} = \boxed{-10}$	Add to eliminate the variable $\boxed{y}$.
$5x = \boxed{-10}$	Simplify and solve for x.
$x = \boxed{-2}$	Divide both sides by $\boxed{5}$ and simplify.

STEP 2 Substitute into one of the original equations and solve for y.

$y = \boxed{3}$ So, $\boxed{(-2, 3)}$ is the solution of the system.

Solve each system of equations by adding or subtracting. (Examples 1, 2)

2. $\begin{cases} x + 2y = -2 \\ -3x + 2y = -10 \end{cases}$ (2, −2)

3. $\begin{cases} 3x + y = 23 \\ 3x - 2y = 8 \end{cases}$ (6, 5)

4. $\begin{cases} -4x - 5y = 7 \\ 3x + 5y = -14 \end{cases}$ (7, −7)

5. $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$ (−3, 8)

6. $\begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases}$ (2, 3)

7. $\begin{cases} -5x + 7y = 11 \\ -5x + 3y = 19 \end{cases}$ (−5, −2)

8. The Green River Freeway has a minimum and a maximum speed limit. Tony drove for 2 hours at the minimum speed limit and 3.5 hours at the maximum limit, a distance of 355 miles. Rae drove 2 hours at the minimum speed limit and 3 hours at the maximum limit, a distance of 320 miles. What are the two speed limits? (Example 3)

a. Write equations to represent Tony's distance and Rae's distance.

Tony: $2x + 3.5y = 355$ Rae: $2x + 3y = 320$

b. Solve the system.

minimum speed limit: 55 mi/h maximum speed limit: 70 mi/h

ESSENTIAL QUESTION CHECK-IN

9. Can you use addition or subtraction to solve any system? Explain.

no; Addition or subtraction can be used only when the coefficients of the x-terms or the y-terms are the same or opposites.

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8.3 LESSON QUIZ

CA CC **8.EE.8b, 8.EE.8c**

Solve each system of equations by adding or subtracting.

1. $\begin{cases} x + 5y = 8 \\ 2x - 5y = 1 \end{cases}$

2. $\begin{cases} 2x + y = 7 \\ -2x - 4y = -16 \end{cases}$

3. $\begin{cases} 3x + 7y = 47 \\ -4x + 7y = 19 \end{cases}$

4. $\begin{cases} x + 3y = -23 \\ -x + 4y = -26 \end{cases}$

5. The perimeter of a rectangle is 24 inches. Twice the length decreased by three times the width is 4 inches. What are the dimensions of the rectangle?

Lesson Quiz available online

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Answers

1. (3, 1)

2. (2, 3)

3. (4, 5)

4. (−2, −7)

5. length = 8 inches; width = 4 inches

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.EE.8b, 8.EE.8c**

Concepts & Skills	Practice
Example 1 Solving a Linear System by Adding	Exercises 1, 4–5
Example 2 Solving a Linear System by Subtracting	Exercises 2–3, 6–7
Example 3 Solving Problems with Systems of Equations	Exercises 8, 10–15

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
10–11	**2** Skills/Concepts	**MP.4** Modeling
12	**2** Skills/Concepts	**MP.2** Reasoning
13–15	**2** Skills/Concepts	**MP.4** Modeling
16	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning
17	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

8.3 Independent Practice

CA CC 8.EE.8b, 8.EE.8c

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10. **Represent Real-World Problems** Marta bought new fish for her home aquarium. She bought 3 guppies and 2 platies for a total of $13.95. Hank also bought guppies and platies for his aquarium. He bought 3 guppies and 4 platies for a total of $18.33. Find the price of a guppy and the price of a platy.

Guppy: $3.19; platy: $2.19

11. **Represent Real-World Problems** The rule for the number of fish in a home aquarium is 1 gallon of water for each inch of fish length. Marta's aquarium holds 13 gallons and Hank's aquarium holds 17 gallons. Based on the number of fish they bought in Exercise 10, how long is a guppy and how long is a platy?

Guppy: 3 in.; platy: 2 in.

12. Line m passes through the points (6, 1) and (2, −3). Line n passes through the points (2, 3) and (5, −6). Find the point of intersection of these lines.

$\left(\frac{7}{2}, -\frac{3}{2}\right)$

13. **Represent Real-World Problems** Two cars got an oil change at the same auto shop. The shop charges customers for each quart of oil plus a flat fee for labor. The oil change for one car required 5 quarts of oil and cost $22.45. The oil change for the other car required 7 quarts of oil and cost $25.45. How much is the labor fee and how much is each quart of oil?

Labor fee: $14.95; quart of oil: $1.50

14. **Represent Real-World Problems** A sales manager noticed that the number of units sold for two T-shirt styles, style A and style B, was the same during June and July. In June, total sales were $2779 for the two styles, with A selling for $15.95 per shirt and B selling for $22.95 per shirt. In July, total sales for the two styles were $2385.10, with A selling at the same price and B selling at a discount of 22% off the June price. How many T-shirts of each style were sold in June and July combined?

280 T-shirts of style A and style B were sold in June and July.

15. **Represent Real-World Problems** Adult tickets to a basketball game cost $5. Student tickets cost $1. A total of $2,874 was collected on the sale of 1,246 tickets. How many of each type of ticket were sold?

407 adult tickets and 839 student tickets

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

16. **Communicate Mathematical Ideas** Is it possible to solve the system $\begin{cases} 3x - 2y = 10 \\ x + 2y = 6 \end{cases}$ by using substitution? If so, explain how. Which method, substitution or elimination, is more efficient? Why?

Yes; solve the second equation for x to get $x = -2y + 6$. Substitute $-2y + 6$ for x in the first equation to get $3(-2y + 6) - 2y = 10$. Solve this for y to get $y = 1$. Then substitute 1 for y in either original equation to get $x = 4$, for a solution of (4, 1). The elimination method is more efficient because there are fewer calculations and they are simpler to do.

17. Jenny used substitution to solve the system $\begin{cases} 2x + y = 8 \\ x - y = 1 \end{cases}$. Her solution is shown below.

Step 1	$y = -2x + 8$	Solve the first equation for y.
Step 2	$2x + (-2x + 8) = 8$	Substitute the value of y in an original equation.
Step 3	$2x - 2x + 8 = 8$	Use the Distributive Property.
Step 4	$8 = 8$	Simplify.

a. **Explain the Error** Explain the error Jenny made. Describe how to correct it.

She substituted her expression for y in the same equation she used to find y. She should substitute her expression into the other equation.

b. **Communicate Mathematical Ideas** Would adding the equations have been a better method for solving the system? If so, explain why.

Yes; adding the equations would have resulted in $3x = 9$, easily giving $x = 3$ after dividing each side by 3. Substitution requires many more steps.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity The solution to a system of three equations in three variables is an ordered triple (x, y, z). You can solve the following system of equations using elimination. Add the first two equations to eliminate z. Add the resulting equation to the third equation to eliminate y. This will give you the value of x, which you can substitute into the third equation to find the value of y. Finally, substitute y in the second equation to find the value of z.

$$\begin{cases} 2x + y + z = 12 \\ 3y - z = -10 \\ x - 4y = 7 \end{cases}$$

(3, −1, 7)

Lesson Support

Content Objective Students will learn how to solve a system of linear equations by elimination with multiplying.

Language Objective Students will give the steps for solving a system of linear equations by elimination with multiplying.

California Common Core Standards

CA CC **8.EE.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

CA CC **8.EE.8c** Solve real-world and mathematical problems leading to two linear equations in two variables.

CA CC **MP.1** Make sense of problems and persevere in solving them.

CA CC **Focus | Coherence | Rigor**

Building Background

Eliciting Prior Knowledge Show students two systems of equations. Discuss the method students would choose to use to solve each system. Students do not have to solve the systems. Instead ask them to focus on how they would select the method they would use and why they would use the method. Solutions are (1, 2) and (3, −1). Elimination by adding would be a good method for the first system; elimination by subtracting would be a good method for the second system. Students might also choose substitution.

$$\begin{cases} 5x + y = 7 \\ -5x - 4y = -13 \end{cases}$$

$$\begin{cases} 3x - 4y = 13 \\ x - 4y = 7 \end{cases}$$

Learning Progressions

In this lesson, students solve a system of two linear equations using the elimination method. Important understandings for students include the following:

- Solve a linear system by multiplying and adding.
- Solve a linear system by multiplying and subtracting.
- Solve problems with systems of linear equations by elimination.

This lesson is a culmination of the methods students have learned to solve a system. They solve linear systems algebraically in which substitution or elimination alone are increasingly difficult to use. They look for appropriate multiples to help them use the elimination method they learned in the last lesson. As with the other lessons in this module, students continue to apply the new method to solving real-life problems arising from a system of linear equations.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Analyze and solve linear equations and pairs of simultaneous linear equations.*** Remind students that they solved equations with rational number coefficients and constants by multiplying by the LCD. Point out that this same method can be used to rewrite the fractions in a system of equations. Ask them to use multiplication and elimination to solve the following system:

$$\begin{cases} \frac{x}{4} + \frac{y}{3} = -6 \\ 2x - 4y = 12 \end{cases}$$

(−12, −9)

Language Support

California ELD Standards

CA ELD **Emerging 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering basic questions with prompting and substantial support.

CA ELD **Expanding 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering detailed questions with occasional prompting and moderate support.

CA ELD **Bridging 2.I.5. Listening actively** – Demonstrate active listening in oral presentation activities by asking and answering detailed questions with minimal prompting and support.

Linguistic Support

Academic/Content Vocabulary

Help students get into the routine of explaining their answer and their process when solving systems. Work collaboratively with English learners to solve Your Turn Exercise 10. As students work through the problem, have another student write down the step-by-step solution so that they can all explain how to solve the problem when it is complete.

Background Knowledge

Encourage students to discuss their answers and their solutions. Structure group work so that each student has a meaningful contribution to make to the whole group task. Use an assignment like this one from the DIFFERENTIATE INSTRUCTION section in the Teacher's Edition.

Leveled Strategies for English Learners

Emerging Have students copy the table in Exercise 13 and draw a symbol to represent the different possibilities (pies and applesauce). Have students work in small groups of mixed English proficiency levels to help formulate an explanation and justification for how to solve the problem.

Expanding Have students form teams to solve Exercise 13. As students solve the problem, have them prepare a statement that tells how they know their answer is correct.

Bridging When students solve Exercise 13, provide them with a sentence frame to write in their math journal how they solved the problem and how they know their answer is correct.

Math Talk To help students answer the question posed in Math Talk, provide sentence frames for them to use.

LESSON

8.4 Solving Systems by Elimination with Multiplication

CA Common Core Standards

The student is expected to:

CA CC Expressions and Equations—8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

CA CC Expressions and Equations—8.EE.8c

Solve real-world and mathematical problems leading to two linear equations in two variables.

Mathematical Practices

CA CC MP.1 Problem Solving

Engage

ESSENTIAL QUESTION

How do you solve a system of linear equations by multiplying? Sample answer: First, decide which variable to eliminate. Then, multiply one equation by a constant so that adding or subtracting will eliminate that variable. Finally, solve the system using the elimination method.

Motivate the Lesson

Ask: Can you solve a system of equations if adding or subtracting does not eliminate one of the variables? Begin the Lesson to find out how you can rewrite one equation and solve the system of equations.

Explore

Have students compare the equations $3x - 5y = -17$ and $6x - 10y = -34$, using graphs and tables. Also have them rewrite both equations in slope-intercept form. Once they see that the equations are the same line, ask them to identify the operation that must be carried out on one equation to make it equal to the other.

Explain

EXAMPLE 1

Questioning Strategies **Mathematical Practices**

- What is the purpose of multiplying one of the equations by a constant? When the equations do not have the same (or opposite) coefficient for one of the variables, then one or both of the equations must be multiplied by a constant in order to create a variable that has the same (or opposite) coefficient. Then, one variable can be eliminated by addition or subtraction.

Focus on Reasoning **Mathematical Practices**

Discuss that the second equation could just as well have been multiplied by -2 and then subtracted, but multiplying by negative numbers and subtracting has a higher chance of leading to errors than multiplying by positive numbers and adding.

YOUR TURN

Avoid Common Errors

Students might not be able to decide which variable to eliminate. Suggest they first check the equations for coefficients that are multiples of each other. If they exist, only one equation need be multiplied before adding or subtracting to eliminate that variable.

ADDITIONAL EXAMPLE 1

Solve the system of equations by multiplying and adding.

$$\begin{cases} 2x + 5y = 8 \\ -x + 3y = 7 \end{cases}$$

$(-1, 2)$

Interactive Whiteboard
Interactive example available online

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LESSON 8.4

Solving Systems by Elimination with Multiplication

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

ESSENTIAL QUESTION

How do you solve a system of linear equations by multiplying?

Solving a System by Multiplying and Adding

In some linear systems, neither variable can be eliminated by adding or subtracting the equations directly. In systems like these, you need to multiply one of the equations by a constant so that adding or subtracting the equations will eliminate one variable. The steps for this method are as follows:

1. Decide which variable to eliminate.
2. Multiply one equation by a constant so that adding or subtracting will eliminate that variable.
3. Solve the system using the elimination method.

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EXAMPLE 1

CA CC 8.EE.8b

Solve the system of equations by multiplying and adding.

$$\begin{cases} 2x + 10y = 2 \\ 3x - 5y = -17 \end{cases}$$

STEP 1 The coefficient of y in the first equation, 10, is 2 times the coefficient of y, 5, in the second equation. Also, the y-term in the first equation is being added, while the y-term in the second equation is being subtracted. To eliminate the y-terms, multiply the second equation by 2 and add this new equation to the first equation.

$2(3x - 5y = -17)$	Multiply each term in the second equation by 2 to get opposite coefficients for the y-terms.
$6x - 10y = -34$	Simplify.
$6x - 10y = -34$	
$+\ 2x + 10y = 2$	Add the first equation to the new equation.
$8x + 0y = -32$	Add to eliminate the variable y.
$8x = -32$	Simplify and solve for x.
$\frac{8x}{8} = \frac{-32}{8}$	Divide each side by 8.
$x = -4$	Simplify.

My Notes

STEP 2 Substitute the solution into one of the original equations and solve for y.

$2x + 10y = 2$	Use the first equation.
$2(-4) + 10y = 2$	Substitute -4 for the variable x.
$-8 + 10y = 2$	Simplify.
$10y = 10$	Add 8 to each side.
$y = 1$	Divide each side by 10 and simplify.

STEP 3 Write the solution as an ordered pair: $(-4, 1)$

STEP 4 Check your answer algebraically.

Substitute -4 for x and 1 for y in the original system.

$$\begin{cases} 2x + 10y = 2 \rightarrow 2(-4) + 10(1) = -8 + 10 = 2 \checkmark \\ 3x - 5y = -17 \rightarrow 3(-4) - 5(1) = -12 - 5 = -17 \checkmark \end{cases}$$

The solution is correct.

Math Talk Mathematical Practices

When you check your answer algebraically, why do you substitute your values for x and y into the original system? Explain.

If there was a multiplication error, the solution might check in the multiplied system even if it were not correct.

Reflect

1. How can you solve this linear system by subtracting? Which is more efficient, adding or subtracting? Explain your reasoning.
   Multiply the second equation by -2 and then subtract. Adding is more efficient because the y-terms already have opposite signs.
2. Can this linear system be solved by adding or subtracting without multiplying? Why or why not?
   No; without multiplying, neither variable will be eliminated.
3. What would you need to multiply the second equation by to eliminate x by adding? Why might you choose to eliminate y instead of x?
   You would need to multiply the second equation by $-\frac{2}{3}$; it is simpler to multiply by the whole number 2.

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YOUR TURN

Solve each system of equations by multiplying and adding.

4. $\begin{cases} 5x + 2y = -10 \\ 3x + 6y = 66 \end{cases}$ $(-8, 15)$

5. $\begin{cases} 4x + 2y = 6 \\ 3x - y = -8 \end{cases}$ $(-1, 5)$

6. $\begin{cases} -6x + 9y = -12 \\ 2x + y = 0 \end{cases}$ $\left(\frac{1}{2}, -1\right)$

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.1

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to make sense of problems and persevere in solving them. Example 3 uses a four-step problem-solving process to determine the number of adult and children tickets purchased. Students analyze the information, formulate a plan, solve the problem, and justify and evaluate the solution.

Math Background

A system of equations can be solved using matrices and row operations. Possible row operations are:

1. Any two rows can be interchanged.
2. Multiply a row by a nonzero constant.
3. Any row can be replaced with the sum of that row and another.

Solve $\begin{cases} 2x + y = 1 \\ 3x - 5y = 8 \end{cases} \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 3 & -5 & 8 \end{bmatrix}$. Multiply row 1 by 5. $\rightarrow \begin{bmatrix} 10 & 5 & 5 \\ 3 & -5 & 8 \end{bmatrix}$. Add rows 1 and 2. $\rightarrow \begin{bmatrix} 13 & 0 & 13 \\ 3 & -5 & 8 \end{bmatrix}$. So $13x = 13$ and $x = 1$.

Using substitution, $y = -1$.

ADDITIONAL EXAMPLE 2

Solve the system of linear equations by multiplying and subtracting.

$$\begin{cases} 4x + y = -8 \\ 2x + 3y = 6 \end{cases}$$

$(-3, 4)$

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Interactive example available online

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EXAMPLE 2

Questioning Strategies Mathematical Practices

- Why must every term in an equation be multiplied by the same number? It is the only way to obtain an equivalent equation.
- What would be different in the solution had the second equation been multiplied by -3? The new equation in Step 1 would have been $-6x + 12y = 78$. This equation would then have been added to the first equation. The solution $(-3, 5)$ would not have changed.

Engage with the Whiteboard

Have a volunteer highlight each original equation in a different color. Have them use the same colors to highlight the locations in which each equation or its equivalent appears in the solution process.

YOUR TURN

Engage with the Whiteboard

In Exercise 9, have a student volunteer show how to multiply one equation to eliminate x. Have another show how to multiply one equation to eliminate y. Point out that both methods result in the same answer.

ADDITIONAL EXAMPLE 3

Meghan needs to board her cats and dogs at a kennel while she is on vacation. Pet Hotel charges \$42.50 for a cat and \$64.00 for a dog for a total cost of \$277.00. Animal Spa charges \$35.50 for a cat and \$50.50 for a dog, for a total cost of \$222.50. How many cats and how many dogs does Meghan have?

2 cats, 3 dogs

Interactive Whiteboard
Interactive example available online

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EXAMPLE 3

Questioning Strategies Mathematical Practices

- How many times can you multiply an equation by a number and still get an equivalent equation? an unlimited number of times.
- How many equations in a system can be multiplied by a number when solving a system? Any number of the equations in a system can be multiplied by a number in order to find a way to eliminate one of the variables.
- How do you decide which power of 10 to use to multiply the equations in Step 2? The greatest decimal place in any of the coefficients or constant terms is hundredths. So multiplying by 100 replaces all of the decimal values with whole numbers.

Focus on Communication

Emphasize the importance of naming the variables being used and keeping track of what they represent. Any pair of variables can be used, but there must be one to represent the number of adults and a different one to represent the number of children.

Solving a System by Multiplying and Subtracting

You can solve some systems of equations by multiplying one equation by a constant and then subtracting.

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EXAMPLE 2

CA CC 8.EE.8b

Solve the system of equations by multiplying and subtracting.

$\begin{cases} 6x + 5y = 7 \\ 2x - 4y = -26 \end{cases}$

STEP 1 Multiply the second equation by 3 and subtract this new equation from the first equation.

$3(2x - 4y = -26)$	Multiply each term in the second equation by 3 to get the same coefficients for the x-terms.
$6x - 12y = -78$	Simplify.
$6x + 5y = 7$ $-(6x - 12y = -78)$	Subtract the new equation from the first equation.
$0x + 17y = 85$	Subtract to eliminate the variable x.
$17y = 85$	Simplify and solve for y.
$\frac{17y}{17} = \frac{85}{17}$	Divide each side by 17.
$y = 5$	Simplify.

STEP 2 Substitute the solution into one of the original equations and solve for x.

$6x + 5y = 7$	Use the first equation.
$6x + 5(5) = 7$	Substitute 5 for the variable y.
$6x + 25 = 7$	Simplify.
$6x = -18$	Subtract 25 from each side.
$x = -3$	Divide each side by 6 and simplify.

STEP 3 Write the solution as an ordered pair: $(-3, 5)$

STEP 4 Check your answer algebraically.
Substitute -3 for x and 5 for y in the original system.

$\begin{cases} 6x + 5y = 7 \rightarrow 6(-3) + 5(5) = -18 + 25 = 7 \checkmark \\ 2x - 4y = -26 \rightarrow 2(-3) - 4(5) = -6 - 20 = -26 \checkmark \end{cases}$

The solution is correct.

My Notes

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YOUR TURN

Solve each system of equations by multiplying and subtracting.

7. $\begin{cases} 3x - 7y = 2 \\ 6x - 9y = 9 \end{cases}$ $(3, 1)$

8. $\begin{cases} -3x + y = 11 \\ 2x + 3y = -11 \end{cases}$ $(-4, -1)$

9. $\begin{cases} 9x + y = 9 \\ 3x - 2y = -11 \end{cases}$ $\left(\frac{1}{3}, 6\right)$

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Solving Problems with Systems of Equations

Many real-world situations can be modeled with a system of equations.

EXAMPLE 3

Problem Solving

CA CC 8.EE.8c

The Simon family attended a concert and visited an art museum. Concert tickets were \$24.75 for adults and \$16.00 for children, for a total cost of \$138.25. Museum tickets were \$8.25 for adults and \$4.50 for children, for a total cost of \$42.75. How many adults and how many children are in the Simon family?

Analyze Information

The answer is the number of adults and children.

Formulate a Plan

Solve a system to find the number of adults and children.

Solve

STEP 1 Choose variables and write a system of equations. Let x represent the number of adults. Let y represent the number of children.

Concert cost: $24.75x + 16.00y = 138.25$
Museum cost: $8.25x + 4.50y = 42.75$

STEP 2 Multiply both equations by 100 to eliminate the decimals.

$100(24.75x + 16.00y = 138.25) \rightarrow 2{,}475x + 1{,}600y = 13{,}825$

$100(8.25x + 4.50y = 42.75) \rightarrow 825x + 450y = 4{,}275$

DIFFERENTIATE INSTRUCTION

Cooperative Learning

Divide students into groups of four and give each group a system of equations, such as the one below. Have one student solve the system by substitution, a second by graphing, a third by elimination with addition, and a fourth by elimination by multiplication and subtraction. Have students discuss their results and the method they prefer.

$\begin{cases} 3x + y = 9 \\ 6x - y = 9 \end{cases}$

$(2, 3)$

Visual Cues

Many students are not very orderly when using elimination with multiplication and subtraction to solve a system. Suggest they write out the steps they are using, and change subtraction to addition. For example:

$2x + y = 1$ $-3x + 5y = -8$	Multiply by 5.	$10x + 5y = 5$ $-3x + 5y = -8$
	Subtract.	$10x + 5y = 5$ $-(-3x + 5y = -8)$
	Add.	$10x + 5y = 5$ $+(3x - 5y = 8$ $13x \quad = 13$

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

YOUR TURN

Engage with the Whiteboard

Have a volunteer write the two equations for the system of equations. Suggest that they make a table to organize the information in the problem. Ask another volunteer to demonstrate how to solve the system.

Talk About It

Check for Understanding

Ask: If nothing else changed, would it have been possible for Seth's average biking speed to be 29.8 mi/h? Justify your answer. It would not be possible. The solution would have been approximately (−1.23, 1), but it is not possible to run for a negative number of hours.

Elaborate

Talk About It

Summarize the Lesson

Ask: How do you know when solving a system of equations that you must multiply before you can add or subtract? If the coefficients of one variable are not the same or are not opposites in the two equations, then you must multiply one or both of the equations until the coefficients of one variable are the same or opposites. Then, you can add or subtract to eliminate one variable.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 1, have volunteers explain the process of arriving at the correct values as they complete the write-in boxes for each step.

Avoid Common Errors

Exercise 4 Remind students that either equation can be selected and then multiplied by a number, and either variable can be eliminated. In this exercise the top equation could be multiplied by 3 so x can be eliminated, or the bottom equation could be multiplied by 2 so y can be eliminated.

Exercise 8 Suggest that students organize the information in this situation in a table and identify what the variables represent. Remind them that eliminating the decimals will make the solution process easier.

STEP 3 Multiply the second equation by 3 and subtract this new equation from the first equation.

$3(825x + 450y = 4{,}275)$	Multiply each term in the second equation by 3 to get the same coefficients for the x-terms.
$2{,}475x + 1{,}350y = 12{,}825$	Simplify.
$2{,}475x + 1{,}600y = 13{,}825$ $-(2{,}475x + 1{,}350y = 12{,}825)$	Subtract the new equation from the first equation.
$0x + 250y = 1{,}000$	Subtract to eliminate the variable x.
$250y = 1{,}000$	Simplify and solve for y.
$\frac{250y}{250} = \frac{1{,}000}{250}$	Divide each side by 250.
$y = 4$	Simplify.

STEP 4 Substitute the solution into one of the original equations and solve for x.

$8.25x + 4.50y = 42.75$	Use the second equation.
$8.25x + 4.50(4) = 42.75$	Substitute 4 for the variable y.
$8.25x + 18 = 42.75$	Simplify.
$8.25x = 24.75$	Subtract 18 from each side.
$x = 3$	Divide each side by 8.25 and simplify.

STEP 5 Write the solution as an ordered pair: (3, 4).
There are 3 adults and 4 children in the family.

Justify and Evaluate

Substituting $x = 3$ and $y = 4$ into the original equations results in true statements. The answer is correct.

My Notes

YOUR TURN

10. Contestants in the Run-and-Bike-a-thon run for a specified length of time, then bike for a specified length of time. Jason ran at an average speed of 5.2 mi/h and biked at an average speed of 20.6 mi/h, going a total of 14.2 miles. Seth ran at an average speed of 10.4 mi/h and biked at an average speed of 18.4 mi/h, going a total of 17 miles. For how long do contestants run and for how long do they bike?

Contestants run 0.75 hour and bike 0.5 hour.

Guided Practice

1. Solve the system $\begin{cases} 3x - y = 8 \\ -2x + 4y = -12 \end{cases}$ by multiplying and adding. (Example 1)

STEP 1 Multiply the first equation by 4. Add to the second equation.

$4(3x - y = 8)$	Multiply each term in the first equation by 4 to get opposite coefficients for the y-terms.
$\boxed{12}x - \boxed{4}y = \boxed{32}$	Simplify.
$+\ (-2x) + \ \ 4y = -12$	Add the second equation to the new equation.
$10x = \boxed{20}$	Add to eliminate the variable $\boxed{y}$.
$x = \boxed{2}$	Divide both sides by $\boxed{10}$ and simplify.

STEP 2 Substitute into one of the original equations and solve for y.

$y = \boxed{-2}$ So, $\boxed{(2, -2)}$ is the solution of the system.

Solve each system of equations by multiplying first. (Examples 1, 2)

2. $\begin{cases} x + 4y = 2 \\ 2x + 5y = 7 \end{cases}$ (6, −1)

3. $\begin{cases} 3x + y = -1 \\ 2x + 3y = 18 \end{cases}$ (−3, 8)

4. $\begin{cases} 2x + 8y = 21 \\ 6x - 4y = 14 \end{cases}$ $\left(\frac{7}{2}, \frac{7}{4}\right)$

5. $\begin{cases} 2x + y = 3 \\ -x + 3y = -12 \end{cases}$ (3, −3)

6. $\begin{cases} 6x + 5y = 19 \\ 2x + 3y = 5 \end{cases}$ (4, −1)

7. $\begin{cases} 2x + 5y = 16 \\ -4x + 3y = 20 \end{cases}$ (−2, 4)

8. Bryce spent \$5.26 on some apples priced at \$0.64 each and some pears priced at \$0.45 each. At another store he could have bought the same number of apples at \$0.32 each and the same number of pears at \$0.39 each, for a total cost of \$3.62. How many apples and how many pears did Bryce buy? (Example 3)

a. Write equations to represent Bryce's expenditures at each store.

First store: $0.64x + 0.45y = 5.26$ Second store: $0.32x + 0.39y = 3.62$

b. Solve the system.

Number of apples: 4 Number of pears: 6

ESSENTIAL QUESTION CHECK-IN

9. When solving a system by multiplying and then adding or subtracting, how do you decide whether to add or subtract?

If the coefficients of one variable are the same, you subtract.

If the coefficients of one variable are opposites, you add.

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8.4 LESSON QUIZ

CA CC **8.EE.8b, 8.EE.8c**

Solve each system of linear equations by multiplying first.

1. $\begin{cases} x + 3y = 8 \\ 2x - 5y = -6 \end{cases}$

2. $\begin{cases} 3x + 2y = 2 \\ -2x - 4y = -12 \end{cases}$

3. $\begin{cases} 2x + 15y = 13 \\ -3x + 5y = 8 \end{cases}$

4. $\begin{cases} x + 2y = -8 \\ -8x + 5y = 1 \end{cases}$

5. The perimeter of a rectangle is 42 feet. The length decreased by three times the width is 1 foot. What are the dimensions of the rectangle?

Lesson Quiz available online

my.hrw.com

Answers

1. (2, 2)
2. (−2, 4)
3. (−1, 1)
4. (−2, −3)
5. length = 16 ft; width = 5 ft

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.EE.8b, 8.EE.8c**

Concepts & Skills	Practice
Example 1 Solving a System by Multiplying and Adding	Exercises 1–7, 10
Example 2 Solving a System by Multiplying and Subtracting	Exercises 2–7
Example 3 Solving Problems with Systems of Equations	Exercises 8, 11–13

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
10	**3** Strategic Thinking H.O.T.	**MP.3** Logic
11	**3** Strategic Thinking H.O.T.	**MP.4** Modeling
12	**2** Skills/Concepts	**MP.2** Reasoning
13	**2** Skills/Concepts	**MP.4** Modeling
14	**3** Strategic Thinking H.O.T.	**MP.3** Logic
15	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

CLUSTER CONNECTION

Exercise 11 combines concepts from the California Common Core cluster "Analyze and solve linear equations and pairs of simultaneous linear equations."

Name _____ Class _____ Date _____

8.4 Independent Practice

CA CC 8.EE.8b, 8.EE.8c

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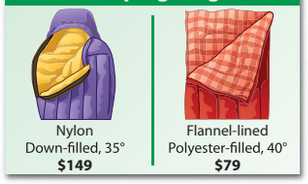
10. Explain the Error Gwen used elimination with multiplication to solve the system $\begin{cases} 2x + 6y = 3 \\ x - 3y = -1 \end{cases}$. Her work to find x is shown. Explain her error. Then solve the system.

$$\begin{aligned} 2(x - 3y) &= -1 \\ 2x - 6y &= -1 \\ +2x + 6y &= 3 \\ 4x + 0y &= 2 \\ x &= \tfrac{1}{2} \end{aligned}$$

Gwen forgot to multiply the right side by 2; $\left(\frac{1}{4}, \frac{5}{12}\right)$

11. Represent Real-World Problems At Raging River Sports, polyester-fill sleeping bags sell for \$79. Down-fill sleeping bags sell for \$149. In one week the store sold 14 sleeping bags for \$1456.

Sleeping Bags



a. Let x represent the number of polyester-fill bags sold and let y represent the number of down-fill bags sold. Write a system of equations you can solve to find the number of each type sold.

$\begin{cases} 79x + 149y = 1456 \\ x + y = 14 \end{cases}$

b. Explain how you can solve the system for y by multiplying and subtracting.

Multiply the second equation by 79. Subtract the new equation from the first one and solve the resulting equation for y.

c. Explain how you can solve the system for y using substitution.

Solve the second equation for x. Substitute the expression for x in the first equation and solve the resulting equation for y.

d. How many of each type of bag were sold?

9 polyester-fill, 5 down-fill

12. Twice a number plus twice a second number is 310. The difference between the numbers is 55. Find the numbers by writing and solving a system of equations. Explain how you solved the system.

105 and 50; Sample answer: I multiplied the second equation in the system $\begin{cases} 2x + 2y = 310 \\ x - y = 55 \end{cases}$ by 2 and then added to eliminate the y-terms.

13. Represent Real-World Problems A farm stand sells apple pies and jars of applesauce. The table shows the number of apples needed to make a pie and a jar of applesauce. Yesterday, the farm picked 169 Granny Smith apples and 95 Golden Delicious apples. How many pies and jars of applesauce can the farm make if every apple is used?

Type of apple	Granny Smith	Golden Delicious
Needed for a pie	5	3
Needed for a jar of applesauce	4	2

21 pies, 16 jars of applesauce

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

14. Make a Conjecture Lena tried to solve a system of linear equations algebraically and in the process found the equation $5 = 9$. Lena thought something was wrong, so she graphed the equations and found that they were parallel lines. Explain what Lena's graph and equation could mean.

Lena's graph shows that the two lines do not intersect. This would seem to mean that the system has no solution. It would seem that solving an equation algebraically and getting a false statement means that the system has no solution.

15. Consider the system $\begin{cases} 2x + 3y = 6 \\ 3x + 7y = -1 \end{cases}$.

a. Communicate Mathematical Ideas Describe how to solve the system by multiplying the first equation by a constant and subtracting. Why would this method be less than ideal?

Multiply the first equation by 1.5 and subtract. This would be less than ideal because you would introduce decimals into the solution process.

b. Draw Conclusions Is it possible to solve the system by multiplying both equations by integer constants? If so, explain how.

Yes; multiply the first equation by 3 and the second equation by 2. Both x-term coefficients would be 6. Solve by eliminating the x-terms using subtraction.

c. Use your answer from part b to solve the system.

$(9, -4)$

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity The solution to a system of three equations in three variables is (x, y, z). Solve the following system of equations using elimination by multiplication and addition or subtraction. Multiply the first equation by 2. Add the first two equations to eliminate z. Add the resulting equation to the third equation, after multiplying the third equation by 4, to eliminate x. This will give you the value of y, which you can substitute into the third equation to find the value of x. Finally, substitute y in the second equation to find the value of z.

$$\begin{cases} 2x + y + 2z = 6 \\ 3y - 4z = -10 \\ x - 4y = 11 \end{cases}$$

$(3, -2, 1)$

Lesson Support

Content Objective Students will learn how to solve systems that have no solution or infinitely many solutions.

Language Objective Students will describe how to identify systems that have no solution or infinitely many solutions.

California Common Core Standards

CA CC **8.EE.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

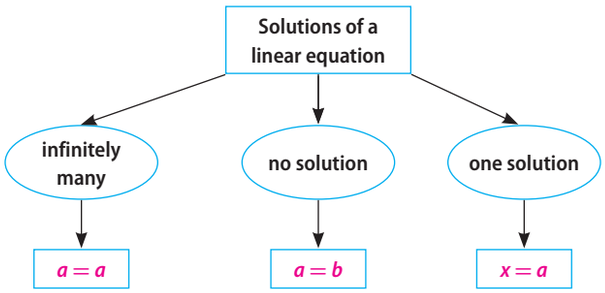
CA CC **8.EE.8c** Solve real-world and mathematical problems leading to two linear equations in two variables.

CA CC **MP.2** Reason abstractly and quantitatively.

CA CC **Focus | Coherence | Rigor**

Building Background

Eliciting Prior Knowledge Have students work with a partner or in small groups to create a case diagram that shows the possible solutions of a linear equation. Encourage students to construct examples for each of the three possible cases in the diagram. Sample completed diagram is shown.



Learning Progressions

In this lesson, students learn that not all systems of linear equations have a unique solution. Students also solve simple cases by inspection. Important understandings for students include the following:

- Solve special systems by graphing.
- Solve special systems algebraically.

In this unit, students have focused on linear equations and, in this module, on systems of linear equations. They have built upon their knowledge of equations from earlier grades and laid the foundation for more extensive study in algebra. This lesson provides a logical conclusion to their persistent work in finding solutions by graphing and algebraically.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in the cluster: ***Analyze and solve linear equations and pairs of simultaneous linear equations.*** Ask students to create three systems of equations, one with infinitely many solutions, one with no solutions, and one with one solution. In each case they should pair the equation $x + y = 5$ with one of the following equations: $x - y = 5$; $2x + 2y = 10$; $x + y = 10$. Discuss with students how they created each system. infinitely many: $2x + 2y = 10$; one: $x - y = 5$; none: $x + y = 10$

Language Support

California ELD Standards

CA ELD **Emerging 2.I.1.** **Exchanging information/ideas** – Contribute to conversations and express ideas by asking and answering yes-no and wh-questions and responding using short phrases.

CA ELD **Expanding 2.I.1.** **Exchanging information/ideas** – Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, asking relevant questions, affirming others, and adding relevant information.

CA ELD **Bridging 2.I.1.** **Exchanging information/ideas** – Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, adding relevant information, building on responses, and providing useful feedback.

Linguistic Support

Academic/Content Vocabulary

In this lesson, students learn that some systems may have no solution or they may have *infinitely many* solutions. Explain that *infinitely many* means the number of solutions in unlimited. Explain that the word *infinite* means "without end." Students who speak Spanish may know the word *fin* means *end* and the word *infinito* is a cognate that means the same as *infinite*.

Background Knowledge

Form small groups of mixed English proficiency levels to work through Your Turn Exercises 6–8. Provide a sentence frame so that students can develop an explanation for their conclusions.

This system has _____ solution(s) because _____.

Leveled Strategies for English Learners

Emerging Using visual representations, such as graphic organizers, is an excellent way to provide comprehension support for students at this English proficiency level. Using visual representations can help them demonstrate their understanding while also developing higher-level thinking. Use the graphic organizer in the DIFFERENTIATE INSTRUCTION box in this lesson.

Expanding Use the graphic organizer in the DIFFERENTIATE INSTRUCTION box in this lesson and have the students at this English proficiency level explain why they placed each element where they did on the graphic organizer.

Bridging Have these students explain to other students why each step in the graphic organizer is placed where it is.

Math Talk To help students answer the question posed in Math Talk, provide students with a sentence frame to respond.

When I solve the system by substitution, I get _____.

The result _____ (does / does not) change the number of solutions because _____.

LESSON

8.5 Solving Special Systems

CA Common Core Standards

The student is expected to:

CA CC Expressions and Equations—8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

CA CC Expressions and Equations—8.EE.8c

Solve real-world and mathematical problems leading to two linear equations in two variables.

Mathematical Practices

CA CC MP.2 Reasoning

Engage

ESSENTIAL QUESTION

How do you solve a system with no solutions or infinitely many solutions? Sample answer: The same methods of graphing, substitution, or elimination are used. If the graph shows parallel lines or if the solution gives a false statement, there is no solution. If the graph shows the lines coincide or if the solution gives a true statement for all ordered pairs, there are infinitely many solutions.

Motivate the Lesson

Ask: Equations whose graphs are intersecting lines have one solution. How many solutions do equations whose graphs are parallel lines have? Take a guess. Begin the Explore Activity to find out.

Explore

EXPLORE ACTIVITY 1

Engage with the Whiteboard

When discussing Step A, have a student highlight each of the equations in different colors and then highlight the corresponding lines in the same colors. When discussing Step B, have a different student highlight the two equations and the corresponding line in the same color.

Explain

EXAMPLE 1

Questioning Strategies **Mathematical Practices**

- How would the results in Step 2 of Part A change if you were to use addition to solve this system? Would the solution change? The results after adding would be $0 = 2$. The solution would still be "no solution."
- How would the results in Step 2 of Part B change if you were to use substitution to solve this system? Would the solution change? The results after substituting would be $0 = 0$ or another true statement. The solution would still be "infinitely many solutions."

Engage with the Whiteboard

For Step 4 of Part A in Example 1, have a volunteer graph the equations on the coordinate grid. Discuss ways to check to be sure the two lines are actually parallel, such as finding whether their slopes are equal. Have another volunteer graph the equations in Step 4 of Part B.

ADDITIONAL EXAMPLE 1

A Solve the system of linear equations by substitution.

$$\begin{cases} x - y = 8 \\ -x + y = 4 \end{cases}$$

B Solve the system of linear equations by elimination.

$$\begin{cases} 3x + 4y = -7 \\ -9x - 12y = 21 \end{cases}$$

A no solution

B infinitely many solutions

Interactive Whiteboard
Interactive example available online

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LESSON 8.5

Solving Special Systems

CA CC 8.EE.8b

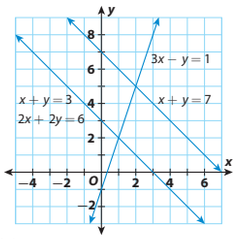
Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

ESSENTIAL QUESTION How do you solve systems with no solution or infinitely many solutions?

EXPLORE ACTIVITY CA CC 8.EE.8b

Solving Special Systems by Graphing

As with linear equations in one variable, some systems may have no solution or infinitely many solutions. One way to tell how many solutions a system has is by inspecting its graph.



Use the graph to solve each system of linear equations.

A $\begin{cases} x + y = 7 \\ 2x + 2y = 6 \end{cases}$

Is there a point of intersection? Explain.

No, the lines appear to be parallel; they have no points in common.

Does this linear system have a solution? Use the graph to explain.

This system has no solution; the lines have no points in common, which means there is no ordered pair that will make both equations true.

B $\begin{cases} 2x + 2y = 6 \\ x + y = 3 \end{cases}$

Is there a point of intersection? Explain.

Yes, the graphs are the same line; all points are points of intersection.

Does this linear system have a solution? Use the graph to explain.

This system has infinitely many solutions; all ordered pairs on the line will make both equations true.

Reflect

1. **Justify Reasoning** Use the graph to identify two lines that represent a linear system with exactly one solution. What are the equations of the lines? Explain your reasoning.

   Sample answer: $x + y = 7$ and $3x - y = 1$; the lines intersect at one point.

EXPLORE ACTIVITY *(cont'd)*

2. A system of linear equations has infinitely many solutions. Does that mean any ordered pair in the coordinate plane is a solution?

   No, only ordered pairs that lie on the graph are solutions.

3. Identify the three possible numbers of solutions for a system of linear equations. Explain when each type of solution occurs.

   one solution when the lines intersect at a single point; no solution when the lines are parallel; infinitely many solutions when the lines are the same line

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Solving Special Systems Algebraically

As with equations, if you solve a system of equations with no solution, you get a false statement, and if you solve a system with infinitely many solutions, you get a true statement.

EXAMPLE 1 CA CC 8.EE.8b

A Solve the system of linear equations by substitution.

$\begin{cases} x - y = -2 \\ -x + y = 4 \end{cases}$

STEP 1 Solve $x - y = -2$ for x:
$x = y - 2$

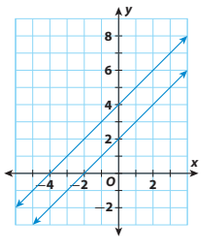
STEP 2 Substitute the resulting expression into the other equation and solve.

$-(y - 2) + y = 4$ *Substitute the expression for the variable x.*

$2 = 4$ *Simplify.*

STEP 3 Interpret the solution. The result is the false statement $2 = 4$, which means there is no solution.

STEP 4 Graph the equations to check your answer. The graphs do not intersect, so there is no solution.



Math Talk Mathematical Practices

The system can be written using $-x + y = 2$ and $-x + y = 4$. How does this show you there is no solution just by looking at it?

The expression $-x + y$ cannot simultaneously equal 2 and 4.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.2

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to reason abstractly and quantitatively. Upon reaching a final algebraic solution of a system of equations, students must reason abstractly to determine whether there is only one solution, no solution, or infinitely many solutions to the system. They must also reason abstractly to analyze the graphs of a system of equations and draw conclusions about the solution(s) of the system.

Math Background

A system of equations is either inconsistent or consistent. An inconsistent system has no solution. A consistent system has at least one solution. A consistent system can be either independent or dependent. An independent system has exactly one solution, and a dependent system has an infinite number of solutions. A dependent consistent system has coefficients and constants that are proportional. For example, $\begin{cases} 2x + 3y = 8 \\ 4x + 6y = 16 \end{cases}$ is dependent and consistent since $\frac{2}{4} = \frac{3}{6} = \frac{8}{16}$.

YOUR TURN

Avoid Common Errors

Remind students that when they multiply an equation by a constant, they must multiply each term in the equation by the constant.

Talk About It

Check for Understanding

Ask: If you were to graph the systems in Exercises 6–8, what would you expect each of the graphs to look like? The graph for the system in Exercise 6 would be parallel lines. The graph for the system in Exercise 7 would be lines intersecting at (10, −2). The graph for the system in Exercise 8 would be one line.

Elaborate

Talk About It

Summarize the Lesson

Ask: How can you tell that a system has one, none, or infinitely many solutions? The graph of a system with one solution is a pair of intersecting lines. The algebraic solution will provide a value for x and a value for y. The graph of a system with no solutions is a pair of parallel lines. The algebraic solution will provide a false statement. The graph of a system with infinitely many solutions is one line. The algebraic solution will provide a true statement.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 1, have volunteers explain their thinking as they fill in the blanks for each step. Ask volunteers to point out or highlight the equations involved as they are referred to in the steps.

Avoid Common Errors

Exercise 3 If students attempt to solve this exercise by elimination, they may incorrectly identify it as having infinitely many solutions, because they will reach a point where they see that the left sides of both equations are identical. Explain that for there to be infinitely many solutions, the right sides of the equations must also be identical.

Exercises 2–4 Suggest that students graph each system on its own coordinate grid and then use the graph to verify their answers.

B Solve the system of linear equations by elimination.

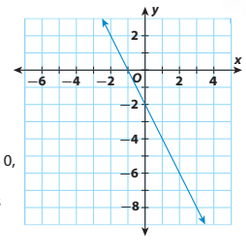
$$\begin{cases} 2x + y = -2 \\ 4x + 2y = -4 \end{cases}$$

STEP 1 Multiply the first equation by -2.

$-2(2x + y = -2) \rightarrow -4x + (-2y) = 4$

STEP 2 Add the new equation from Step 1 to the original second equation.

$$\begin{array}{rrl} & -4x + (-2y) = & 4 \\ + & 4x + \quad 2y = & -4 \\ \hline & 0x + \quad 0y = & 0 \\ & 0 = & 0 \end{array}$$



STEP 3 Interpret the solution. The result is the statement $0 = 0$, which is always true. This means that the system has infinitely many solutions.

STEP 4 Graph the equations to check your answer. The graphs are the same line, so there are infinitely many solutions.

Math Talk Mathematical Practices

What solution do you get when you solve the system in part B by substitution? Does this result change the number of solutions? Explain.

$-4 = -4$; no, this is still a true statement, so the system still has infinitely many solutions.

Reflect

4. If x represents a variable and a and b represent constants so that $a \neq b$, interpret what each result means when solving a system of equations.

$x = a$ The system has one solution.

$a = b$ The system has no solution.

$a = a$ The system has infinitely many solutions.

5. **Draw Conclusions** In part B, can you tell without solving that the system has infinitely many solutions? If so, how?

Yes; the corresponding constants and coefficients of the second equation are twice those of the first equation. Both equations represent the same line, so there are infinitely many solutions.

YOUR TURN

Solve each system. Tell how many solutions each system has.

6. $\begin{cases} 4x - 6y = 9 \\ -2x + 3y = 4 \end{cases}$ no solution

7. $\begin{cases} x + 2y = 6 \\ 2x - 3y = 26 \end{cases}$ (10, −2); one solution

8. $\begin{cases} 12x - 8y = -4 \\ -3x + 2y = 1 \end{cases}$ infinitely many solutions

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Guided Practice

1. Use the graph to find the number of solutions of each system of linear equations. (Explore Activity)

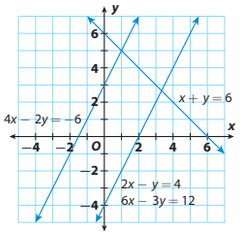
A. $\begin{cases} 4x - 2y = -6 \\ 2x - y = 4 \end{cases}$ **B.** $\begin{cases} 4x - 2y = -6 \\ x + y = 6 \end{cases}$ **C.** $\begin{cases} 2x - y = 4 \\ 6x - 3y = 12 \end{cases}$

STEP 1 Decide if the graphs of the equations in each system intersect, are parallel, or are the same line.

System A: The graphs are parallel.

System B: The graphs intersect.

System C: The graphs are the same line.



STEP 2 Use the results of Step 1 to decide how many points the graphs have in common.

System A has no point(s) in common.

System B has 1 point(s) in common.

System C has infinitely many point(s) in common.

Solve each system. Tell how many solutions each system has. (Example 1)

2. $\begin{cases} x - 3y = 4 \\ -5x + 15y = -20 \end{cases}$ infinitely many solutions

3. $\begin{cases} 6x + 2y = -4 \\ 3x + y = 4 \end{cases}$ no solution

4. $\begin{cases} 6x - 2y = -10 \\ 3x + 4y = -25 \end{cases}$ (−3, −4); one solution

Find the number of solutions of each system without solving algebraically or graphing. Explain your reasoning. (Example 1)

5. $\begin{cases} -6x + 2y = -8 \\ 3x - y = 4 \end{cases}$ infinitely many solutions; they are equations of the same line, $3x - y = 4$.

6. $\begin{cases} 8 = 7x + 2y \\ 7x + 2y = 4 \end{cases}$ no solution; $7x + 2y$ cannot equal 4 and 8 at the same time.

ESSENTIAL QUESTION CHECK-IN

7. When you solve a system of equations algebraically, how can you tell whether the system has zero, one, or an infinite number of solutions?

If your solution gives specific values for x and y, the system has one solution, (x, y). If it gives a false statement, there is no solution. If it gives a true statement, there are infinitely many solutions.

DIFFERENTIATE INSTRUCTION

Cooperative Learning

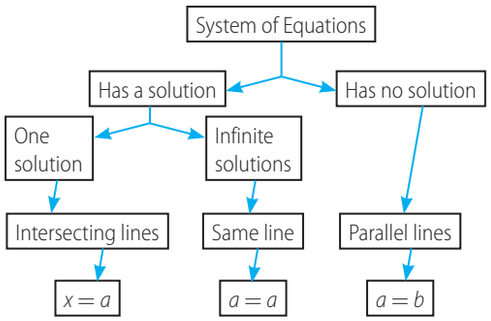
Have students work in groups of three. Have each student roll a number cube to determine a value for the variable n in the system below. Have students determine if the systems created have one, none, or an infinite number of solutions.

$$\begin{cases} 4x + 2y = 10 \\ 2x + y = n \end{cases}$$

When $n = 5$, there are an infinite number of solutions; for all other values of n, there are no solutions.

Graphic Organizer

Have students create a concept map to show the connections between different types of systems of two equations. For example:



Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer
Online Assessment and Intervention

Online homework assignment available

my.hrw.com

8.5 LESSON QUIZ

CA CC **8.EE.8b, 8.EE.8c**

Solve each system. Tell how many solutions each system has.

1. $\begin{cases} x + 3y = 1 \\ 2x + 6y = 2 \end{cases}$

2. $\begin{cases} 3x - 2y = 7 \\ -2x - 4y = 6 \end{cases}$

3. $\begin{cases} 5x + 3y = -2 \\ -15x - 9y = 5 \end{cases}$

Solve each system. Tell how many solutions each system has. Describe the graph of each system.

4. $\begin{cases} 5x + 2y = 3 \\ -5x - 2y = 3 \end{cases}$

5. $\begin{cases} 3x + 6y = 3 \\ -x - 2y = -1 \end{cases}$

6. $\begin{cases} x - 7y = 15 \\ -2x - 5y = -11 \end{cases}$

Lesson Quiz available online

my.hrw.com

Answers

1. infinitely many solutions
2. (1, −2); one solution
3. no solution
4. no solution; parallel lines
5. infinitely many solutions; one line
6. (8, −1); intersecting lines

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **8.EE.8b, 8.EE.8c**

Concepts & Skills	Practice
Explore Activity Solving Special Systems by Graphing	Exercises 1, 8–9, 16–17
Example 1 Solving Special Systems Algebraically	Exercises 2–4, 5–6, 10–15, 18–19

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
8–9	**1** Recall	**MP.2** Reasoning
10–16	**2** Skills/Concepts	**MP.2** Reasoning
17	**3** Strategic Thinking H.O.T.	**MP.3** Logic
18–19	**3** Strategic Thinking H.O.T.	**MP.4** Modeling
20–21	**3** Strategic Thinking H.O.T.	**MP.3** Logic
22	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

8.5 Independent Practice

CA CC 8.EE.8b, 8.EE.8c

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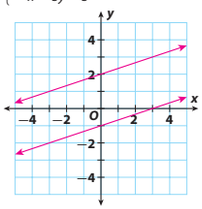
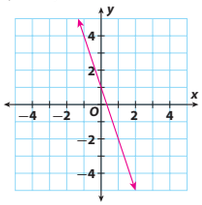
Solve each system by graphing. Check your answer algebraically.

8. $\begin{cases} -2x + 6y = 12 \\ x - 3y = 3 \end{cases}$



Solution: no solution

9. $\begin{cases} 15x + 5y = 5 \\ 3x + y = 1 \end{cases}$



Solution: infinitely many solutions

For Exs. 10–16, state the number of solutions for each system of linear equations.

10. a system whose graphs have the same slope but different *y*-intercepts
no solution

11. a system whose graphs have the same *y*-intercepts but different slopes
one solution

12. a system whose graphs have the same *y*-intercepts and the same slopes
infinitely many solutions

13. a system whose graphs have different *y*-intercepts and different slopes
one solution

14. the system $\begin{cases} y = 2 \\ y = -3 \end{cases}$ no solution

15. the system $\begin{cases} x = 2 \\ y = -3 \end{cases}$ one solution

16. the system whose graphs were drawn using these tables of values:

Equation 1

x	0	1	2	3
y	1	3	5	7

Equation 2

x	0	1	2	3
y	3	5	7	9

no solution

17. **Draw Conclusions** The graph of a linear system appears in a textbook. You can see that the lines do not intersect on the graph, but also they do not appear to be parallel. Can you conclude that the system has no solution? Explain.
No; although the lines do not intersect on the graph, they intersect at a point that is not on the graph. To prove that a system has no solution, you must do so algebraically.

18. **Represent Real-World Problems** Two school groups go to a roller skating rink. One group pays \$243 for 36 admissions and 21 skate rentals. The other group pays \$81 for 12 admissions and 7 skate rentals. Let *x* represent the cost of admission and let *y* represent the cost of a skate rental. Is there enough information to find values for *x* and *y*? Explain.
No; there are infinitely many solutions to the system.

19. **Represent Real-World Problems** Juan and Tory are practicing for a track meet. They start their practice runs at the same point, but Tory starts 1 minute after Juan. Both run at a speed of 704 feet per minute. Does Tory catch up to Juan? Explain.
No; both Juan and Tory run at the same rate, so the lines representing the distances each has run are parallel. There is no solution to the system.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

20. **Justify Reasoning** A linear system with no solution consists of the equation $y = 4x - 3$ and a second equation of the form $y = mx + b$. What can you say about the values of *m* and *b*? Explain your reasoning.
$m = 4$ and $b \neq -3$; The graphs of the lines must be parallel and thus must have the same slope, so $m = 4$. The *y*-intercepts must be different because two equations with the same slope and the same *y*-intercept are the same line, so $b \neq -3$.

21. **Justify Reasoning** A linear system with infinitely many solutions consists of the equation $3x + 5 = 8$ and a second equation of the form $Ax + By = C$. What can you say about the values of *A*, *B*, and *C*? Explain your reasoning.
A, *B*, and *C* must all be the same multiple of 3, 5, and 8, respectively. The two equations represent a single line, so the coefficients and constants of one equation must be a multiple of the other.

22. **Draw Conclusions** Both the points (2, −2) and (4, −4) are solutions of a system of linear equations. What conclusions can you make about the equations and their graphs?
The linear system has more than one solution, so the lines coincide. There are infinitely many solutions.

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity For each of the following systems, find the value or values for *a* and *b* that make the system have no solution.

1) $\begin{cases} 3x - y = -4 \\ y = ax + b \end{cases}$ **2)** $\begin{cases} -x + ay = 0 \\ -2x + 8y = b \end{cases}$ **3)** $\begin{cases} x = a \\ y = b \end{cases}$

1) *a* must have a value of 3; *b* can have any value except 4.

2) *a* must have a value of 4; *b* can have any value except 0.

3) This system will have exactly one solution for any possible values of *a* and *b*.

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI Response to Intervention

Intervention	Enrichment
Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.	

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Online and Print Resources

Differentiated Instruction
- Reteach worksheets
- Reading Strategies EL
- Success for English Learners EL

Differentiated Instruction
- Challenge worksheets PRE-AP

Extend the Math PRE-AP
Lesson Activities in TE

Additional Resources

Assessment Resources includes:
- Leveled Module Quizzes

MODULE QUIZ

Ready to Go On?

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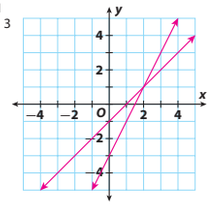
8.1 Solving Systems of Linear Equations by Graphing

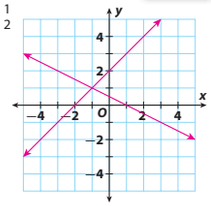
Solve each system by graphing.

1. $\begin{cases} y = x - 1 \\ y = 2x - 3 \end{cases}$



(2, 1)

2. $\begin{cases} x + 2y = 1 \\ -x + y = 2 \end{cases}$



(−1, 1)

8.2 Solving Systems by Substitution

Solve each system of equations by substitution.

3. $\begin{cases} y = 2x \\ x + y = -9 \end{cases}$ (−3, −6)

4. $\begin{cases} 3x - 2y = 11 \\ x + 2y = 9 \end{cases}$ (5, 2)

8.3 Solving Systems by Elimination

Solve each system of equations by adding or subtracting.

5. $\begin{cases} 3x + y = 9 \\ 2x + y = 5 \end{cases}$ (4, −3)

6. $\begin{cases} -x - 2y = 4 \\ 3x + 2y = 4 \end{cases}$ (4, −4)

8.4 Solving Systems by Elimination with Multiplication

Solve each system of equations by multiplying first.

7. $\begin{cases} x + 3y = -2 \\ 3x + 4y = -1 \end{cases}$ (1, −1)

8. $\begin{cases} 2x + 8y = 22 \\ 3x - 2y = 5 \end{cases}$ (3, 2)

8.5 Solving Special Systems

Solve each system. Tell how many solutions each system has.

9. $\begin{cases} -2x + 8y = 5 \\ x - 4y = -3 \end{cases}$ no solution

10. $\begin{cases} 6x + 18y = -12 \\ x + 3y = -2 \end{cases}$ infinitely many

ESSENTIAL QUESTION

11. What are the possible solutions to a system of linear equations, and what do they represent graphically?
No solution: parallel lines; one solution: intersecting lines; infinitely many solutions: same line

California Common Core Standards

Lesson	Exercises	Common Core Standards
8.1	1–2	8.EE.8a, 8.EE.8c
8.2	3–4	8.EE.8b, 8.EE.8c
8.3	5–6	8.EE.8b, 8.EE.8c
8.4	7–8	8.EE.8b, 8.EE.8c
8.5	9–10	8.EE.8b, 8.EE.8c

Assessment Readiness

Scoring Guide

Item 3 Award the student 1 point for solving the system to find the numbers of daisies and tulips and 1 point for justifying the choice of solution method.

Item 4 Award the student 1 point for writing and solving a system of equations to find the cost of the bus passes and 1 point for explaining how to check the solution.

Additional Resources

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Online Assessment and Intervention
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To assign this assessment online, login to your Assignment Manager at **my.hrw.com**.

CALIFORNIA

MODULE 8
MIXED REVIEW

Assessment Readiness

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1. Consider each system of equations. Does the system have at least one solution? Select Yes or No for systems A–C.

A. $\begin{cases} 3x - 2y = 4 \\ -6x + 4y = -8 \end{cases}$ ● Yes ○ No

B. $\begin{cases} -4x + y = 1 \\ 12x - 3y = 3 \end{cases}$ ○ Yes ● No

C. $\begin{cases} 2x + y = 0 \\ 4x - y = -6 \end{cases}$ ● Yes ○ No

2. Ruby ran 5 laps on the inside lane around a track. Dana ran 3 laps around the same lane on the same track, and then she ran 0.5 mile to her house. Both girls ran the same number of miles in all. The equation $5d = 3d + 0.5$ models this situation.

Choose True or False for each statement.

A. The variable d represents the distance around the track in miles. ● True ○ False

B. The expression $5d$ represents the time it took Ruby to run 5 laps. ○ True ● False

C. The expression $3d + 0.5$ represents the distance in miles Dana ran. ● True ○ False

3. Daisies cost \$0.99 each and tulips cost \$1.15 each. Maria bought a bouquet of daisies and tulips. It contained 12 flowers and cost \$12.52. Solve the system $\begin{cases} x + y = 12 \\ 0.99x + 1.15y = 12.52 \end{cases}$ to find the number of daisies x and the number of tulips y in Maria's bouquet. State the method you used to solve the system and why you chose that method.

(8, 4), so the bouquet had 8 daisies and 4 tulips; sample answer: I used substitution to solve the system because the first equation is easy to solve for y.

4. Kylie bought 4 daily bus passes and 2 weekly bus passes for \$29.00. Luis bought 7 daily bus passes and 2 weekly bus passes for \$36.50. Write and solve a system of equations to find the cost of a daily pass and the cost of a weekly pass. Explain how you can check your answer.

$\begin{cases} 4x + 2y = 29.00 \\ 7x + 2y = 36.50 \end{cases}$, where x is the cost of a daily pass and y is the cost of a weekly pass; (2.50, 9.50), so a daily pass costs \$2.50, and a weekly pass costs \$9.50; sample answer: Substitute the values into each equation to see if they make each one true.

California Common Core Standards

Items	Grade 8 Standards	Mathematical Practices
1	8.EE.8, 8.EE.8b	MP.2
2*	8.EE.7b	MP.2
3	8.EE.8, 8.EE.8b, 8.EE.8c	MP.1, MP.2
4	8.EE.8, 8.EE.8b, 8.EE.8c	MP.2, MP.4

* Item integrates mixed review concepts from previous modules or a previous course.

CLUSTER CONNECTION

Item 4 combines concepts from the California Common Core cluster "Analyze and solve linear equations and pairs of simultaneous linear equations."